Scientific Philosophy

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Philosophy of Quantum Mechanics

Quantum Mechanics (QM) is a fundamental physical theory about atomic-scale processes. It was formulated in the first decades of the XX century by many of the most distinguished physicists of that time. The accordance of this theory with experimental results is remarkable. The physical interpretation of the different mathematical constructs that appear in the formalism of QM, however, raised unprecedented controversies.

The referents of QM are particular physical systems called ***quantum systems***. The states of a quantum system are represented by a non-unique, normalized, mathematical function called ***wave function***, defined on a Euclidean 3-dimensional space. The wave function belongs to an abstract infinite-dimensional complex functional space called the ***Hilbert space***.

$$\psi(\overline{x}) \in H$$

The values of properties of a quantum system can be calculated with ***self-adjoint operators*** *Â(t)* acting upon the corresponding wave functions. But, unlike classical systems, quantum systems may not have precise values for its properties. Instead, we can calculate the average of a certain property by

$$(\forall \mathcal{A} \in \mathcal{P})\,(\exists \hat{A} \in A \,/\, \hat{A} \hat{=} \mathcal{A}).$$

$$\widehat{A}(t) : H \longrightarrow H,$$

$$\left\langle \widehat{A} \right\rangle = \left\langle \psi | \widehat{A} | \psi \right\rangle.$$

$$\langle \psi | \phi \rangle = \int d\overline{x}\, \psi^{*}(\overline{x}) \cdot \phi(\overline{x}).$$

The ***spread*** of the average of a given property is

$$\Delta_\psi \widehat{A}^2 = \left\langle \widehat{A} \right\rangle^2 - \left\langle \widehat{A}^2 \right\rangle.$$

If the spread is zero, the property is ***sharp.***

$$\widehat{A}\psi_k(\overline{x}) = \lambda_k \psi_k(\overline{x}).$$

Under certain conditions, the values *k* may constitute a countable set, i.e. the values of the property may be ***quantized***.

For given an eigenstate of certain self-adjoint operator $\hat{A}(t)$, the propensity p_k of any quantum state to take the value k is

$$p_k = |\langle\psi|\psi_k\rangle|^2,$$

with $0<p_k<1$.

QM has an evolution equation that describes how properties change with time:

H denotes a particular operator called Hamiltonian of the system.

$$\frac{d\widehat{A}}{dt} = \frac{i}{\hbar}(\widehat{H}\widehat{A} - \widehat{A}\widehat{H}) + \frac{\partial\widehat{A}}{\partial t},$$

Heisenberg's equation

An alternative, equivalent formulation of the theory can be obtained adopting time-independent operators to represent the properties and a time-dependent wave function that obeys the ***Schrödinger's equation:***

$$\widehat{H}|\psi(x)>=\frac{i}{\hbar}\frac{\partial|\psi(x)>}{\partial t}.$$

Since the evolution equations are linear the ***Superposition Principle*** holds: any linear combination of solutions is solution.

Axiomatic foundations of QM
(Perez-Bergliaffa, Romero, & Vucetich 1993, 1996)

GROUP I: SPACE AND TIME

A_1 $E_3 \equiv$ tridimensional euclidean space.

A_2 $E_3 \hat{=}$ physical space. (SA)

A_3 $T \equiv$ interval of the real line R.

A_4 $T \hat{=}$ time interval. (SA)

A_5 The relation $\leqslant$ that orders T means "before to" $\vee$ "simultaneous with". (SA)

GROUP II: MICROSYSTEMS AND STATES

A_6 $\Sigma, \overline{\Sigma}$: non-empty, denumerable sets.

A_7 $\forall \sigma \in \Sigma, \sigma$ denotes a microsystem. In particular, σ_0 denotes absence of microsystem. (SA)

A_8 $\forall \overline{\sigma} \in \overline{\Sigma}$, $\overline{\sigma}$ denotes environment of some system. In particular, $\overline{\sigma}_0$ denotes the empty environment, $< \sigma, \overline{\sigma}_0 >$ denotes a free microsystem, and $< \sigma_0, \overline{\sigma}_0 >$ denotes the vacuum. (SA)

A_9 $\forall < \sigma, \overline{\sigma} > \in \Sigma \times \overline{\Sigma}$, $\exists \mathcal{H}_e / \mathcal{H}_e = < \mathcal{S}, \mathcal{H}, \mathcal{S}' > \equiv$ equipped Hilbert space.

A_{10} There exists a one-to-one correspondence between physical states of $\sigma \in \Sigma$ and rays $\Psi \subset \mathcal{H}$. (SA)

GROUP III: OPERATORS AND PHYSICAL QUANTITIES

$\mathbf{A_{11}}$ $\mathcal{P} \equiv$ non-empty family of functions on Σ.

$\mathbf{A_{12}}$ $\mathrm{A} \equiv$ ring of operators on $\mathcal{H}_e$.

$\mathbf{A_{13}}$ $\forall \mathcal{A} \in \mathcal{P}$, $\mathcal{A}$ designates a property of $\sigma \in \Sigma$. (SA)

$\mathbf{A_{14}}$ $(\forall \mathcal{A} \in \mathcal{P})\,(\exists \hat{A} \in \mathrm{A} / \hat{A} \hat{=} \mathcal{A})$. (SA)

$\mathbf{A_{15}}$ (Hermiticity and linearity)
$(\forall \sigma \in \Sigma) \wedge (\forall t / t = t_0 \text{ with } t_0 \text{ fixed}) \wedge (\forall \hat{A} \in \mathrm{A} / \hat{A} \hat{=} \mathcal{A},\ \mathcal{A} \in \mathcal{P})$
if $|\psi_1>, |\psi_2> \in \mathcal{H}_e \Rightarrow$

1. $\hat{A} : \mathcal{H}_e \to \mathcal{H}_e / \hat{A}[\lambda_1|\psi_1> + \lambda_2|\psi_2>] = \lambda_1 \hat{A}|\psi_1> + \lambda_2 \hat{A}|\psi_2>$
   with $\lambda_1, \lambda_2 \in \mathcal{C}$
2. $\hat{A}^\dagger = \hat{A}$ on $\mathcal{H}$.

A_{16} (Probability densities)
$(\forall <\sigma, \overline{\sigma}> \in \Sigma \times \overline{\Sigma}) \wedge (\forall \hat{A} \in A / \hat{A} \hat{=} \mathcal{A}, \mathcal{A} \in \mathcal{P}) \wedge (\forall |a> \in \mathcal{H} / \hat{A}|a>= a|a>) \wedge (\forall |\psi> \in \Psi \subset \mathcal{H}$ that corresponds to the state of σ when it is influenced by $\overline{\sigma}$):

$<\psi|a><a|\psi> \equiv$ probability density for the property $\mathcal{A}$ when σ is associated to $\overline{\sigma}$
(i.e. $\int_{a_1}^{a_2} <\psi|a><a|\psi> \, da$ is the probability for σ to have an $\mathcal{A}$-value in $[a_1, a_2]$). (SA)

A_{17} $(\forall\, \sigma \in \Sigma) \wedge (\forall\, \overline{\sigma} \in \overline{\Sigma})$ the ray Ψ corresponding to a state of σ is the null ray on the border of the accesible region for the system $\sigma \dotplus \overline{\sigma}$.

A_{18} $(\forall\, \sigma \in \Sigma) \wedge (\forall \hat{A} \in A\,) \wedge (\forall a\, / eiv\, \hat{A} = a)$ a is the sole value that $\mathcal{A}$ takes on σ, given that $\hat{A} \hat{=} \mathcal{A}$. (SA)

A_{19} $\hbar \in \mathcal{R}^{+}$.

A_{20} $[\hbar] = LMT^{-1}$.

GROUP IV: SYMMETRIES AND GROUP STRUCTURE

A_{21} (Unitary operators)
$(\forall < \sigma, \overline{\sigma} > \in \Sigma \times \overline{\Sigma}) \wedge (\forall \hat{A} \in A / \hat{A} \hat{=} \mathcal{A}, \mathcal{A} \in \mathcal{P})$ if $\exists \hat{U} / \hat{U}^{\dagger} = \hat{U}^{-1} \Rightarrow \hat{A}' = \hat{U}^{\dagger} \hat{A} \hat{U} \hat{=} \mathcal{A}$. (SA)

A_{22} $\forall < \sigma, \overline{\sigma}_0 > \in \Sigma \times \overline{\Sigma} \; \exists \; \hat{D}(\tilde{G})$, unitary ray representation of some central non-trivial extension of the universal covering group $\bar{G}$ of a Lie group G by a one-dimensional abelian group on $\mathcal{H}$.

A_{23} The Lie algebra $\mathcal{G}$ of the group G is generated by $\{\hat{H}, \hat{P}_i, \hat{K}_i, \hat{J}_i\} \subset A$.

$\mathbf{A_{24}}$ (Algebra structure)
The structure of $\tilde{\mathcal{G}}$, Lie algebra of $\tilde{G}$ is:

$$[\hat{J}_i, \hat{J}_j] = i\hbar\epsilon_{ijk}\hat{J}_k \qquad [\hat{J}_i, \hat{K}_j] = i\hbar\epsilon_{ijk}\hat{K}_k \qquad [\hat{J}_i, \hat{P}_j] = i\hbar\epsilon_{ijk}\hat{P}_k$$

$$[\hat{K}_i, \hat{H}] = i\hbar\hat{P}_i \qquad [\hat{K}_i, \hat{P}_j] = i\hbar\delta_{ij}\hat{M}$$

$$[\hat{J}_i, \hat{H}] = 0 \quad [\hat{K}_i, \hat{K}_j] = 0 \quad [\hat{P}_i, \hat{P}_j] = 0 \quad [\hat{P}_j, \hat{H}] = 0$$

$$[\hat{J}_i, \hat{M}] = 0 \quad [\hat{K}_i, \hat{M}] = 0 \quad [\hat{P}_i, \hat{M}] = 0 \quad [\hat{H}, \hat{M}] = 0$$

where $\hat{M}$ is an element of the Lie algebra of a one-parameter subgroup (which is used to extend $\overline{G}$).

$\mathbf{A_{25}}$ G is the Galilei group.

A_{26} $\hat{H}$ is the time-translations generator.

A_{27} $\forall <\sigma, \overline{\sigma}> \in \Sigma \times \overline{\Sigma}$, *eiv* $\hat{H} = E$ represents the energy value of σ when it is influenced by $\overline{\sigma}$. (SA)

A_{28} $\hat{P}_i$ is the generator of spatial translations on the cartesian coordinate axis X_i.

A_{29} $\forall <\sigma, \overline{\sigma}> \in \Sigma \times \overline{\Sigma}$, *eiv* $\hat{P}_i = p_i$ represents the i-component of the linear momentum of σ. (SA)

A_{30} $\hat{J}_i$ is the generator of spatial rotations around the Cartesian coordinate axis X_i.

A_{31} $\forall <\sigma, \overline{\sigma}> \in \Sigma \times \overline{\Sigma}$, *eiv* $\hat{J}_i = j_i$ represents the i-component of the angular momentum of σ. (SA)

A_{32} $\hat{K}_i$ is the generator of pure transformations of Galilei on the axis X_i.

A_{33} $\hat{M}$ has a discrete spectrum of real and positive eigenvalues.

A_{34} $\forall <\sigma, \overline{\sigma}> \in \Sigma \times \overline{\Sigma}$, *eiv* $\hat{M} = \mu$ represents the mass of σ. (SA)

A_{35} $\forall <\sigma, \overline{\sigma}> \in \Sigma \times \overline{\Sigma}$, if $\hat{X}_i =_{Df} \frac{1}{\mu}\hat{K}_i$, then *eiv* $\hat{X}_i = x_i$ represents the i-component of the position of σ. (SA)

GROUP V: GAUGE TRANSFORMATIONS AND ELECTRIC CHARGE

A_{36} $(\forall <\sigma,\overline{\sigma}> \in \Sigma \times \overline{\Sigma})\, \exists\, \hat{Q} \in A\, /(\hat{Q} \neq \hat{I}) \wedge ([\hat{Q},\hat{A}] = 0\ \forall \hat{A} \in A)$.

A_{37} $\hat{Q}$ has a discrete spectrum of real eigenvalues.

A_{38} $\hat{Q}$ is the generator of gauge transformations of the first kind.

A_{39} $\forall <\sigma,\overline{\sigma}> \in \Sigma \times \overline{\Sigma}$, *eiv* $\hat{Q} = q$ represents the charge of σ. (SA)

A_{40} There exists one and only one normalized state with *eiv* $\hat{Q} = 0$, named the neutral state.

A_{41} There exists one and only one normalizable state, named vacuum, that is invariant under $\hat{D}(\tilde{G})$ and under gauge transformations of the first kind.

A_{42} If $\sigma \in \Sigma$, *eiv* $\hat{M} = \mu \neq 0$, *eiv* $\hat{Q} = e$ and $<A_0, \vec{A}>$ are the components of an electromagnetic quadripotential that represents the action of $\overline{\sigma} \neq \overline{\sigma}_0$ on $\sigma \Rightarrow$

$$\hat{H} = \frac{1}{2\mu}(\hat{\vec{P}} - \frac{e}{c}\vec{A})^2 + \frac{e}{c}A_0 - g_l \frac{\hbar e}{mc}\vec{B}.\hat{\vec{\sigma}}$$

where $\vec{B}$ has the usual meaning that follows from **P_{10}**, $\hat{\vec{\sigma}}$ is specified in **T_{13}** and g_l is the gyromagnetic factor of the microsystem.

Philosophical issues

Observers: No reference to observers in the axiomatic base. Then, observers cannot appear in the theorems. The reference class is formed by quantum systems and the objects of the background theories.

Heisenberg's inequalities:

$(\forall < \sigma, \overline{\sigma} > \in \Sigma \times \overline{\Sigma}) \wedge (\forall \ |\psi> \in \mathcal{H}) \wedge (\forall \ \{\hat{A}, \hat{B}, \hat{C}\} \subset A$
$/\hat{A} \triangleq \mathcal{A}, \ \hat{B} \triangleq \mathcal{B}, \ \hat{C} \triangleq \mathcal{C}$ with $\{\mathcal{A}, \ \mathcal{B}, \ \mathcal{C}\} \subset \mathcal{P})$ if $[\hat{A}, \ \hat{B}] = i\hat{C} \Rightarrow$

$$(\Delta\hat{A})^2(\Delta\hat{B})^2 \geqslant |\hat{C}|^2/4.$$

Corollary: If $[\hat{X}_i, \hat{P}_j] = \hbar\,\delta_{ij}\,\hat{I}$ then

$$\Delta\hat{X}_i\,\Delta\hat{P}_j \geqslant \hbar/2.$$

Time is not an operator!

$(\forall <\sigma,\overline{\sigma}>\in \Sigma\times\overline{\Sigma}) \wedge (\forall\,|\psi>\in \mathcal{H}) \wedge (\forall\hat{A} \in A\ /\hat{A}\hat{=}\mathcal{A}$ with $\mathcal{A} \in \mathcal{P}) \wedge (\forall\hat{H}/[\hat{H},\ \hat{A}] = i\hat{C})$:

$$\Delta\hat{H}\,\tau_A \geqslant \frac{\hbar}{2}$$

with $\tau_A = \Delta\hat{A}/|d<\hat{A}>/dt|$.

Collapse

A semantical axiom that usually appears in the standard formulation of the theory is the so-called von Neumman's projection postulate:

"If the measurement of a physical observable *A* (with associated operator Â) on a quantum system in the state |j> gives a real value a_n, then, immediately after the measurement, the system evolves from the state $|j_n>$, where $\hat{A}\,|j_n> = a_n\,|j_n>$."

This postulate interprets the collapse of the wave function as a consequence of the act of measuring the property *A*. In our formulation of QM this postulate plays no role.

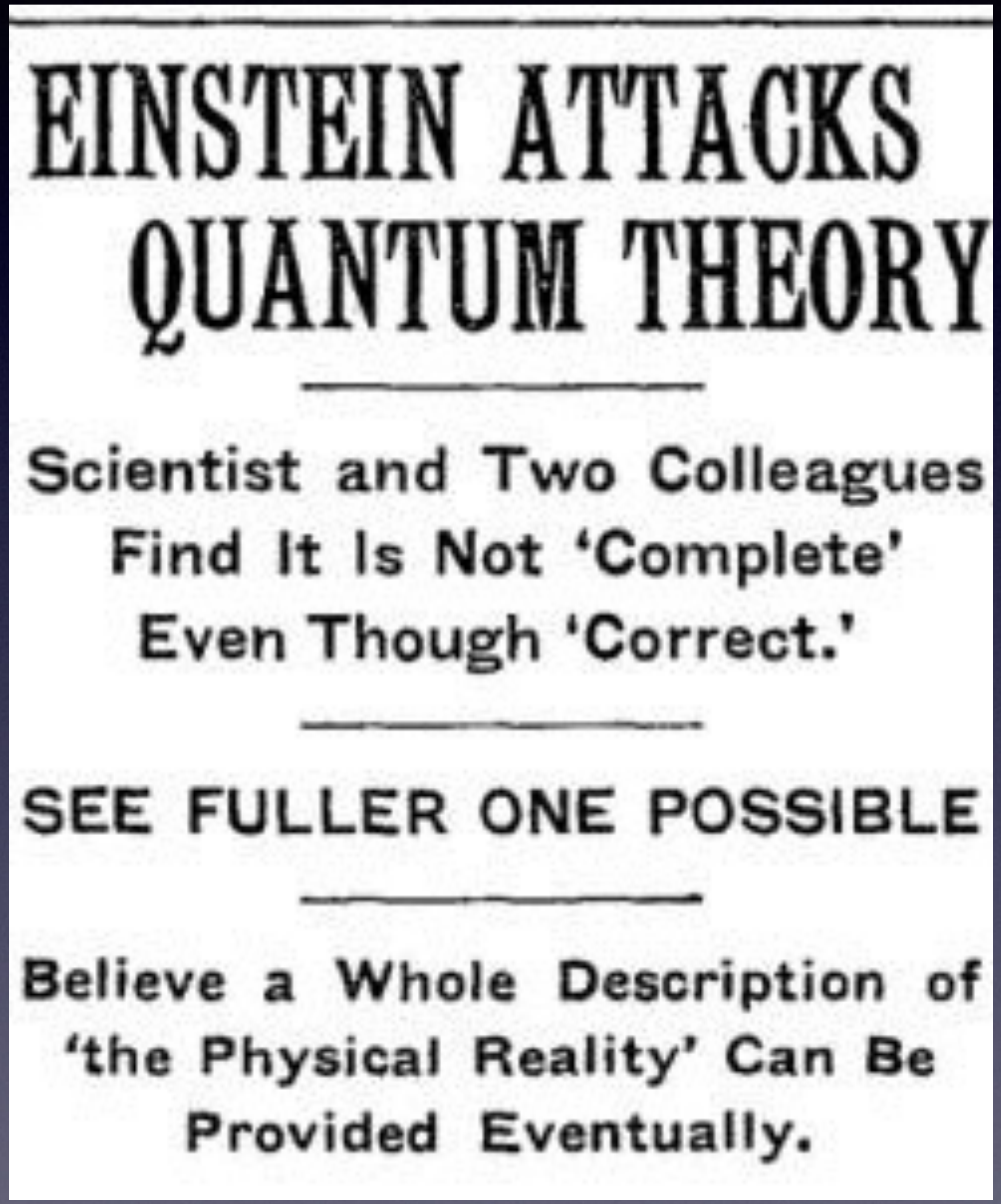
There is no “collapse” of the wave function in QM. The evolution of the system after an interaction is surely non-linear and should be described by a quantum theory of interactions with macro systems. Such a theory is not general, but dependent on the modelling of the macro system.

Notice that mathematical functions do not collapse: they are constructs, not things.

EPR

Albert Einstein, Boris Podolsky and Nathan Rosen proposed an imaginary experiment with the aim of verifying the foundations of Quantum Mechanics. It is today known as the EPR experiment, also known as EPR Paradox.

The physicist Alain Aspect made an experiment based on the idea of the EPR experiment, and supposedly the non-local principle was confirmed by his experiment.


EINSTEIN ATTACKS QUANTUM THEORY

Scientist and Two Colleagues Find It Is Not 'Complete' Even Though 'Correct.'

SEE FULLER ONE POSSIBLE

Believe a Whole Description of 'the Physical Reality' Can Be Provided Eventually.

THE INITIAL HYPOTHESES OF THE EPR EXPERIMENT

These were as follows:

a) The foresights of Quantum Mechanics are righteous

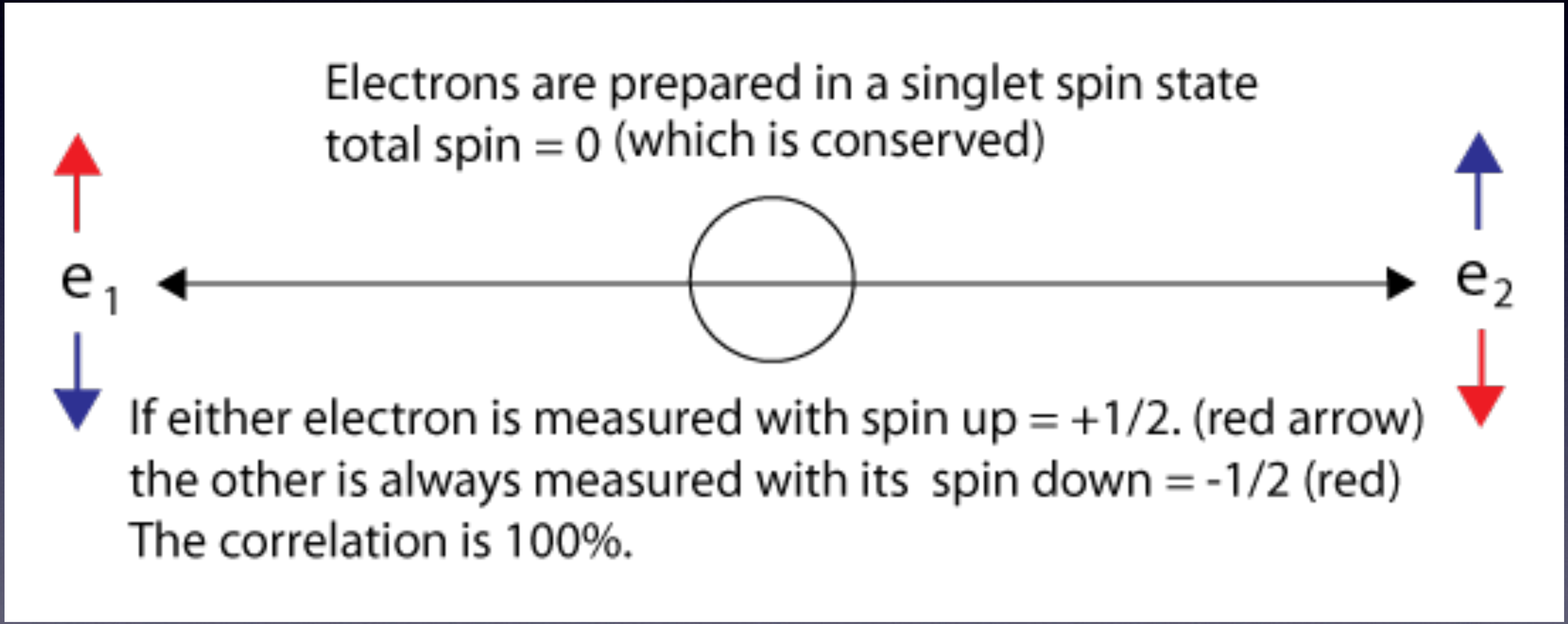
b) No influence can propagate faster than light,

c) If, without disturbing a system in any way, we can foresee with certainty the value of a physical quantity, then there is an element of physical reality that corresponds to this quantity.

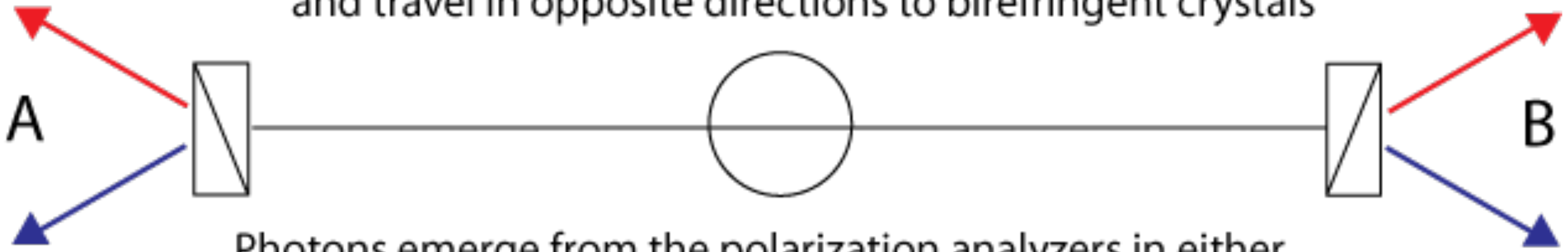
According to the EPR experiment, if the fundamental background of Quantum Mechanics is correct, that would have to imply the existence of non-local interactions in Nature. This means that two particles could “interact” instantaneously without a signal being transmitted from one to another. In another words, if Quantum Mechanics is correct, this implies that Nature works by non-local interactions.

The physicist John Bell proposed a theorem, known as Bell’s inequality, according to which any theory that attempts to describe reality and which, by satisfying the hypotheses “a” and “c”, necessarily violates hypothesis “b”, must to be non-local. This theorem implies that theories based on hidden variables must be discarded.

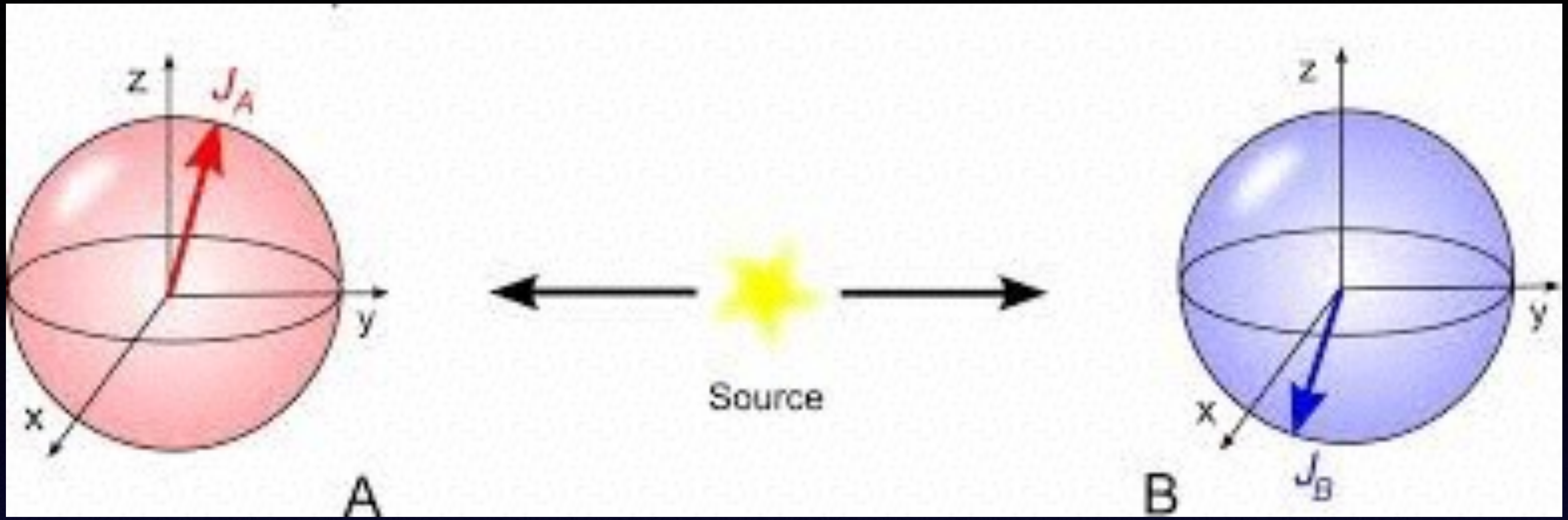
Later Alain Aspect performed an experiment that supposedly confirmed Bell’s theorem. Aspect used photons and, according to the interpretation of the experiment, two photons had instantaneous interaction without having any signal transmitted between them.

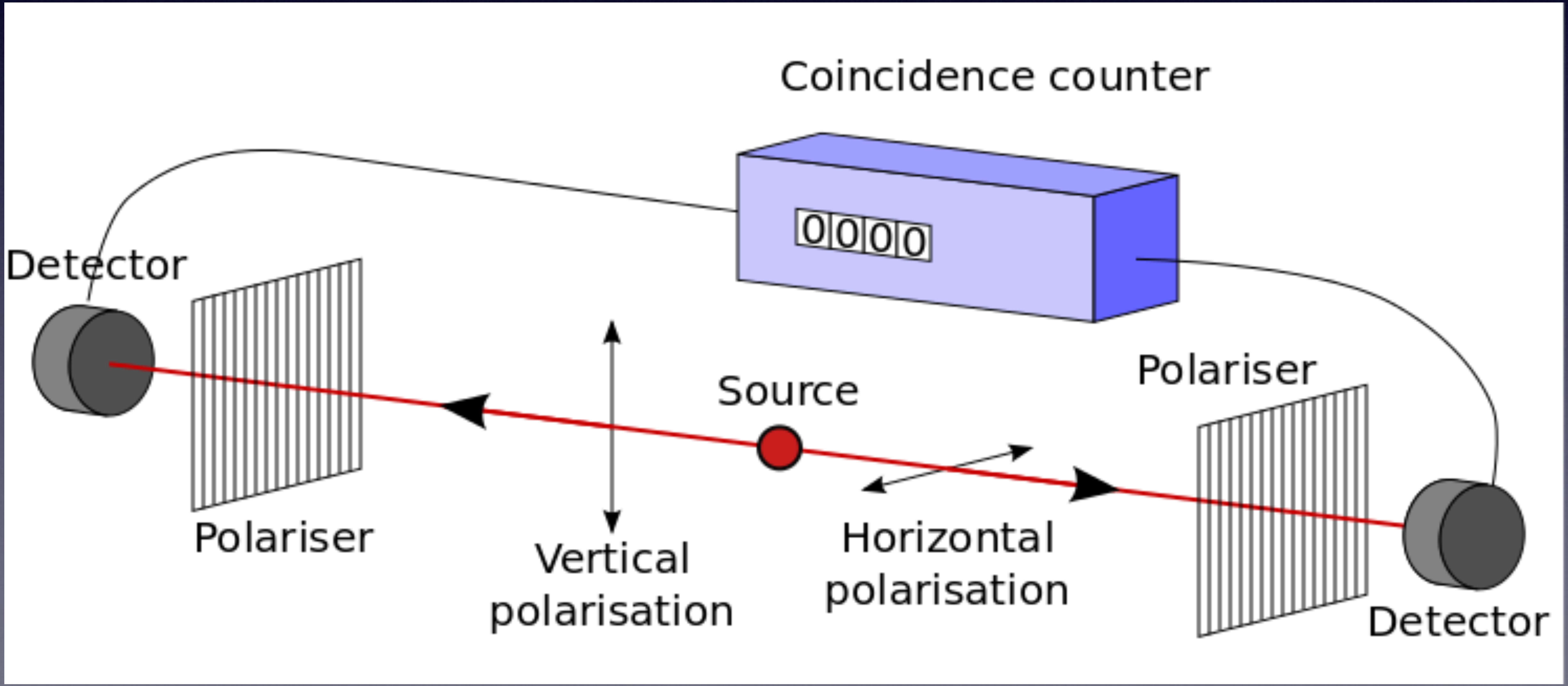

Electrons are prepared in a singlet spin state
total spin = 0 (which is conserved)
e 1
e 2
If either electron is measured with spin up = +1/2. (red arrow)
the other is always measured with its spin down = -1/2 (red)
The correlation is 100%.

Photons are prepared with opposite circular polarizations
and travel in opposite directions to birefringent crystals



Photons emerge from the polarization analyzers in either
the ordinary ray (red arrow) or the extraordinary ray (blue arrow).
Photons that were entangled at the source again show 100%
correlations when measured independently at A and B.


z
J_A
y
x
A
Source
z
y
x
J_B
B


Coincidence counter
0000
Detector
Polariser
Source
Vertical polarisation
Horizontal polarisation
Polariser
Detector

Correlation ≠ interaction

Interaction requires a change of state, not its specification. What is no local are not interactions but systemic correlations. Hence, there is no propagation of superluminal signal. There is no “quantum force”.

But the fact that QM is non-local does not imply that Nature is non-local.

Summing up: QM can be interpreted in a **realistic** and **objective** way. There is not such a thing as a collapse of a wave function, but a **non-linear evolution** of the physical system, that does not follows the linear equations of QM.

Heisenberg's inequalities do not require observers or physical interactions. They refer to **objective properties of quantum systems**.

EPR paradox and the refutation of Bell's inequalities do not imply a breakdown of realism. The Aspect experiments only show that there are apparently non-local correlations. Correlations are not interactions.

Bell's inequality, Bell's theorem

$$\begin{tikzcd} X_1 \arrow[r, dash] \arrow[d, dash] & Y_1 \arrow[d, dash] \\ Y_2 \arrow[r, dash] & X_2 \end{tikzcd}$$

$$X_1 = Y_2 \ \ \& \ \ Y_2 = X_2 \ \ \& \ \ X_2 = Y_1 \ \ \Rightarrow \ \ X_1 = Y_1$$

$$\therefore \ \boxed{X_1 \neq Y_1 \ \Rightarrow \ X_1 \neq Y_2 \ \text{ or } \ Y_2 \neq X_2 \ \text{ or } \ X_2 \neq Y_1}$$

$$\therefore \ \mathrm{P}(X_1 \neq Y_1) \leq \mathrm{P}(X_1 \neq Y_2) + \mathrm{P}(Y_2 \neq X_2) + \mathrm{P}(X_2 \neq Y_1)$$

$$\therefore \ \boxed{\mathrm{P}_{11}(X \neq Y) \leq \mathrm{P}_{12}(X \neq Y) + \mathrm{P}_{22}(Y \neq X) + \mathrm{P}_{21}(X \neq Y)}$$

where $\mathrm{P}_{ab}(\ldots) = \mathrm{P}(\ \ldots \mid A = a,\ B = b)$

NB:(probabilistic) Bell inequality is actually just a simple corollary of a <u>logical implication</u>