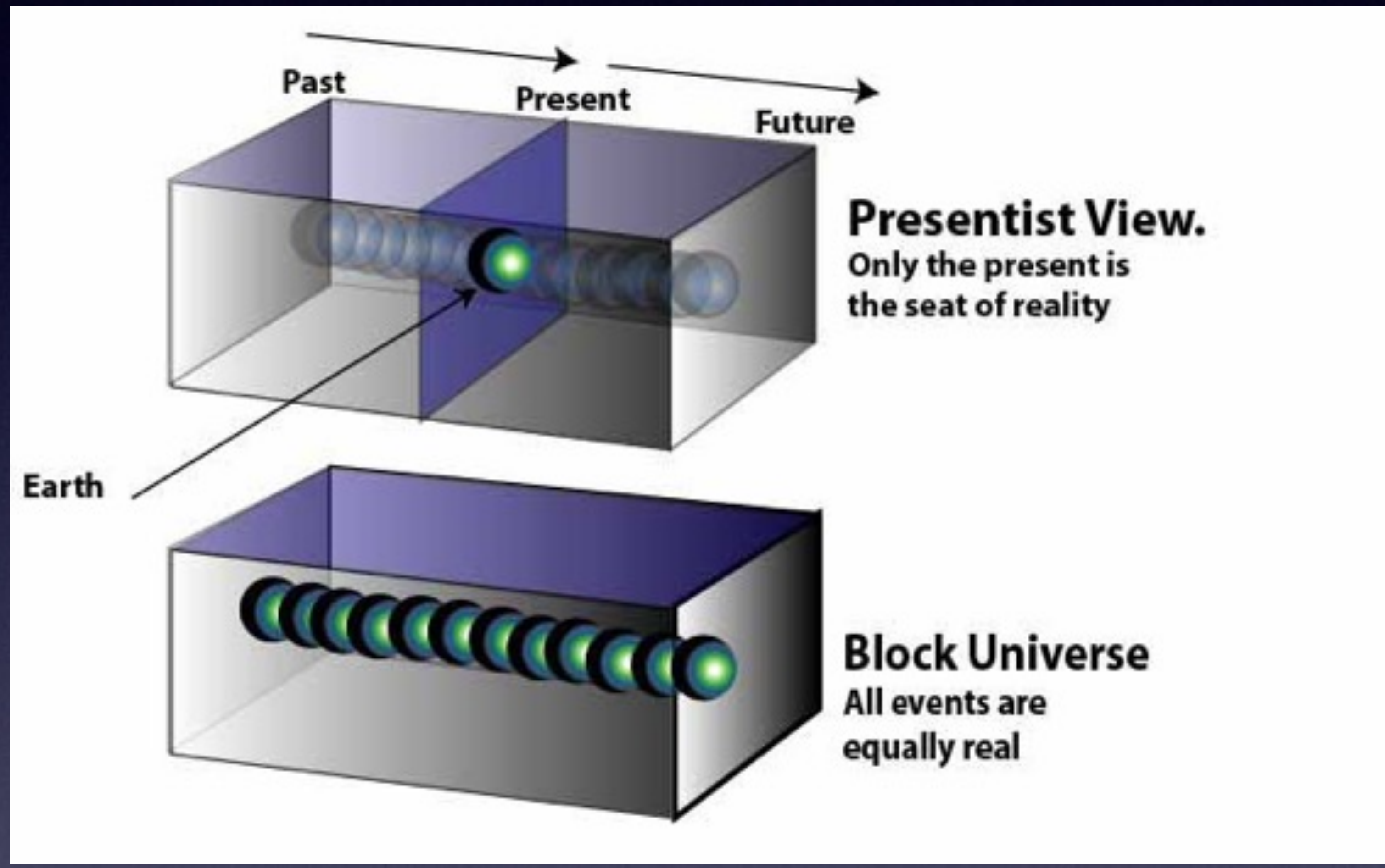
Event ontology and the foundations of space-time



Gustavo E. Romero

IAR-CONICET/UNLP, Argentina

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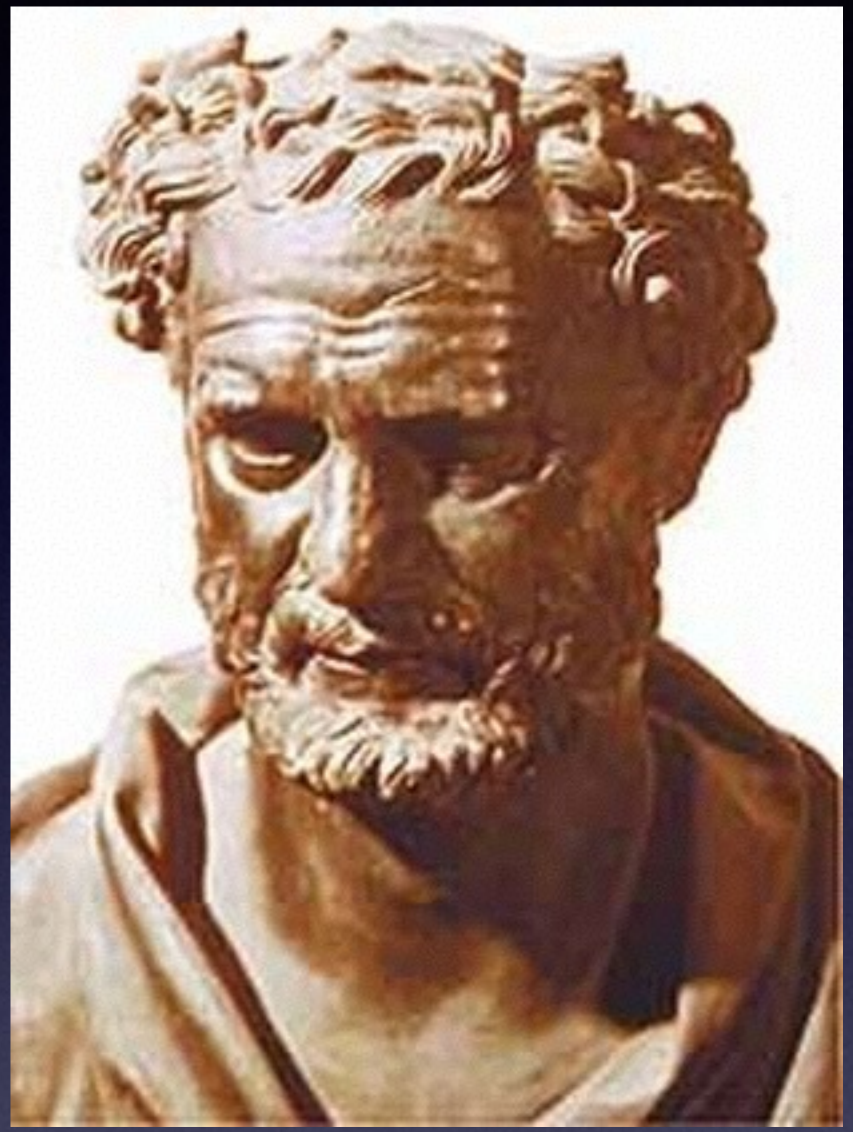
Event ontology in the West

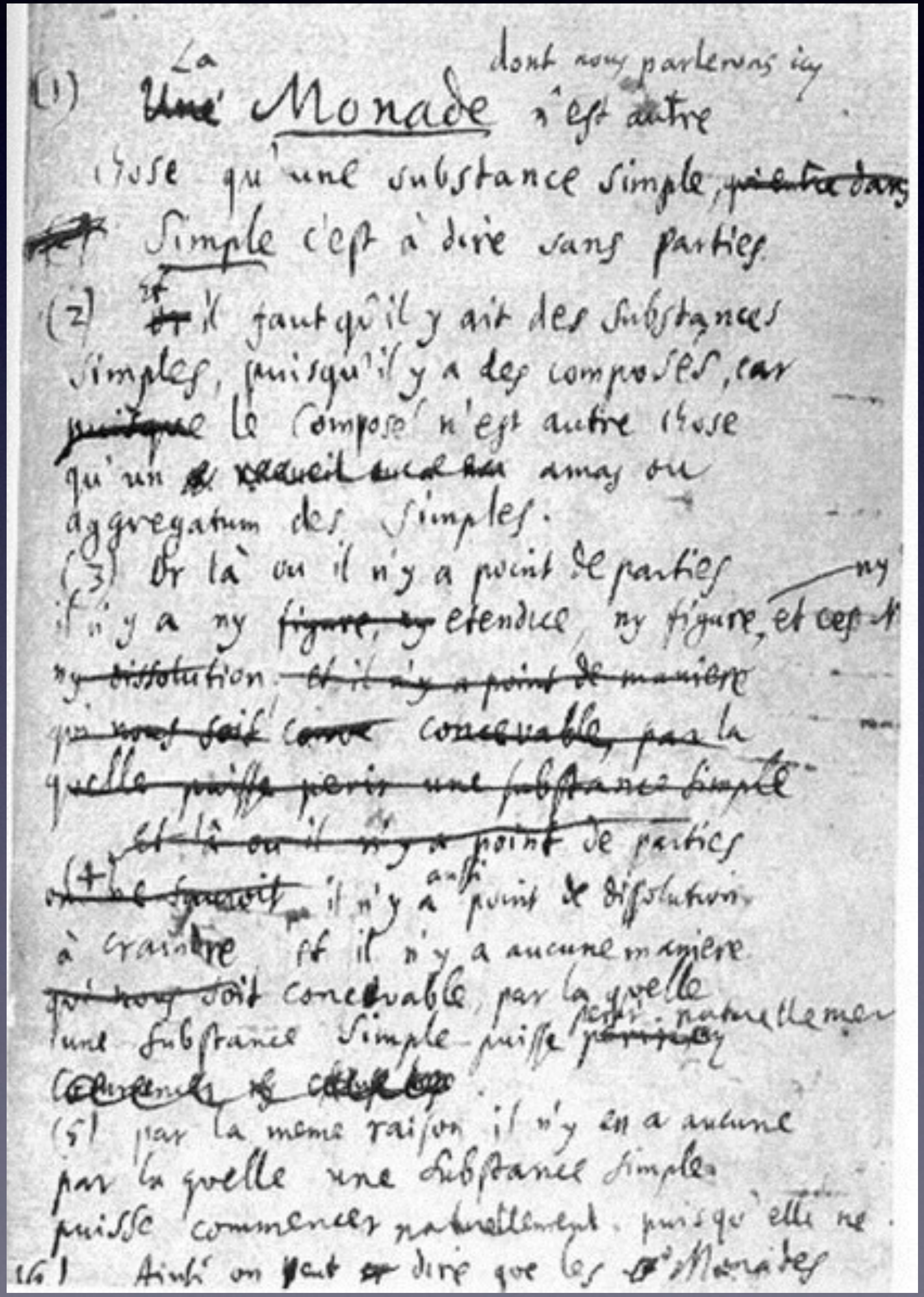
- Events have been usually interpreted in the West as changes in things.
- Just few Western thinkers saw events as fundamental entities: Heraclitus, Cratylus, and Leibniz.



Leibniz and the Monadology

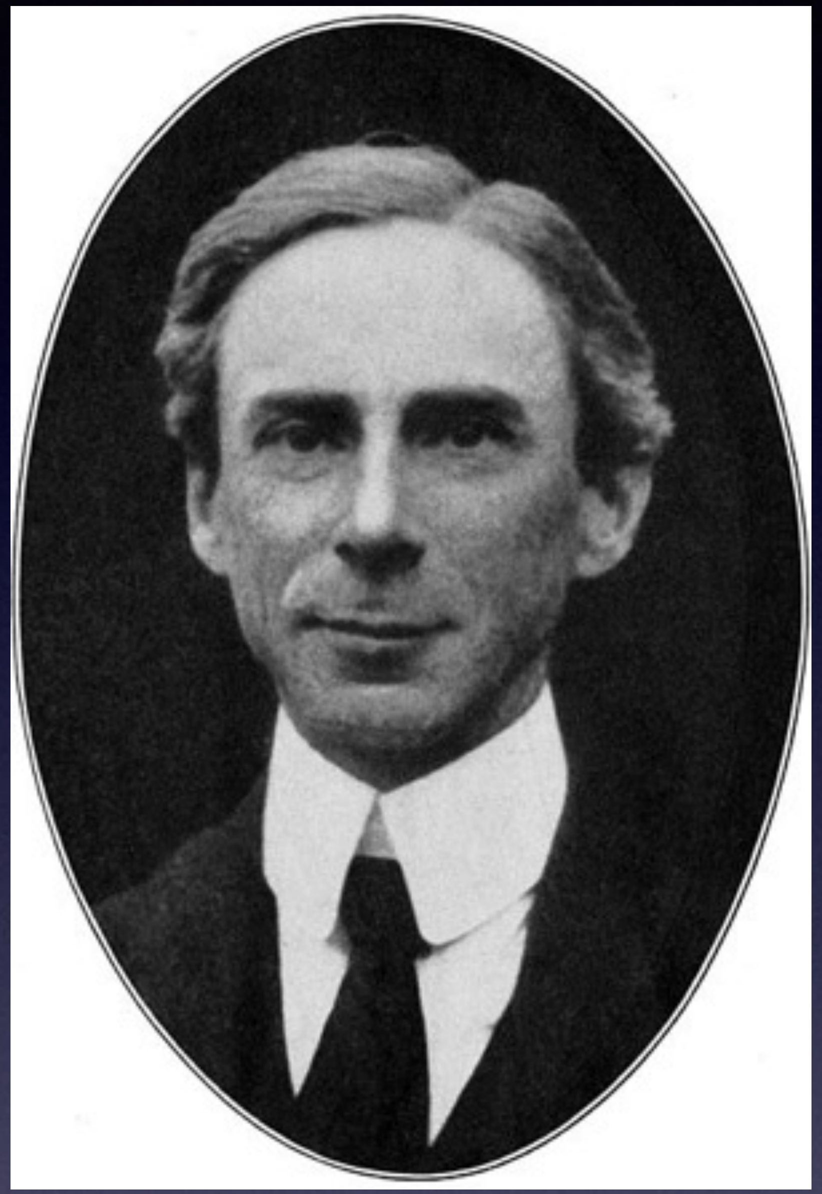
- In his posthumous work *The Monadology,* Leibniz proposed an ontological system in which the basic components of the world were “indivisible centers of actions”, the so-called *monads*.

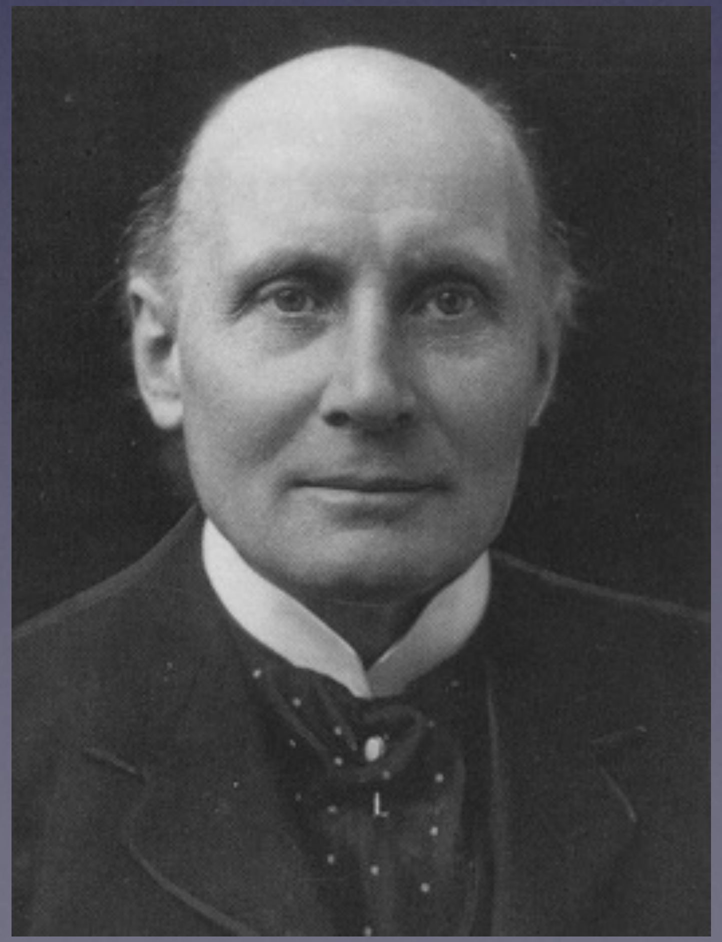
- These monads can be interpreted as basic events (Rescher 1996) and Leibniz’s the first hipothetico-deductive ontological system for events.


La
dont nous parlerons icy
(1) Une Monade n'est autre
chose qu'une substance simple,
Simple c'est à dire sans parties
(2) Il faut qu'il y ait des substances
simples, puisqu'il y a des composés, car
le composé n'est autre chose
qu'un amas ou
aggregatum des simples.
(3) Or là ou il n'y a point de parties
il n'y a ny etendue, ny figure, et
ny dissolution.
concevable, par la
quelle puisse perir une substance simple
et là ou il n'y a point de parties
(4) il n'y a point de dissolution
à craindre et il n'y a aucune maniere
soit concevable, par laquelle
une substance simple puisse perir naturellement
(5) par la meme raison il n'y en a aucune
par laquelle une substance simple
puisse commencer naturellement, puisqu'elle ne
(6) Ainsi on peut dire que les Monades

Russell and Whitehead

- Russell was strongly influenced by Leibniz and in 1900 he published *A Critical Exposition of the Philosophy of Leibniz.*
- Then, in 1914, after finishing *Principia Mathematica*, in his *Our Knowledge of the External World,* Russell considered objects as a complex of phenomena.
- Alfred North Whitehead went far beyond and tried to interpret the world as a complex of processes in his *Process and Reality* (1929). He was strongly influenced by Hegel doctrines. The basic entities of his ontology are the "occasions of experience".





Minkowski and the World

- In 1908 Minkowski proposed a new interpretation of Special Relativity and introduced the concept of space-time.

- Minkowski's space-time is formed by the totality of *events.* This totality he calls "the World". Events are primitives, and the history of things are strings of events.

- Physical laws can be understood as restrictions on the event-space. Hence, Minkowski approach was fundamental for the original development of General Relativity in the period 1912-1915.

- General Relativity is a theory about the structure of space-time, but it says nothing about its ontological nature.

Outline of a theory of events

I assume the existence of events. I consider events as basic individuals. I will formulate here the outlines of theory about events using first order standard logic. I assume also set theory and all mathematics that can be obtained from set theory.

The primitive basis is:

$$\mathcal{B} =< E, \mathcal{E}, e^0, \star >,$$

where E is a set, $\mathcal{E}$ is the collection of all events, e^0 is a fiction called the null event, and $\star$ is a binary operation on E. The meaning of all these symbols will become clear through a set of axioms.

Some axioms

- P_1. $(\forall e)_E\ (e \star e = e)$.
- P_2. $(\forall e_1)_E(\forall e_2)_E\ (e_1 \star e_2 \in E)$.
- P_3. $(\forall x)_{\mathcal{E}}(\exists e)_E\ (e \triangleq x)$.
- P_4. $(\forall x)_{\mathcal{E}}\ (e_1 \triangleq x \wedge e_2 \triangleq x) \Rightarrow (e_1 = e_2)$.
- P_5. $(\exists e^0)(\forall e)_E\ (e^0 \star e = e \star e^0 \equiv e)$.
- P_6. $\neg(\exists x)_{\mathcal{E}}\ (e^0 \triangleq x)$.
- P_7. Card $(E) < \aleph_0$.

Some definitions and theorems

- D$_1$. An event $e_1 \in E$ is composed $\Leftrightarrow (\exists e_2, e_3)_E\,(e_1 = e_2 \star e_3)$
- D$_2$. An event $e_1 \in E$ is basic $\Leftrightarrow\ \neg(\exists e_2, e_3)_E\,(e_1 = e_2 \star e_3)$
- D$_3$. $e_1 \subset e_2\ \Leftrightarrow e_1 \star e_2 = e_2\ \ (e_1$ is part of $e_2\ \Leftrightarrow e_1 \star e_2 = e_2)$
- D$_4$. $\mathrm{Comp}(e) \equiv \{e_i \in E \mid e_i \subset e\}$ is the composition of e.
- D$_5$. $E^0 = E \cup \{e^0\}$.

The following theorems are immediate:

$\vdash (\forall e)_E\,(e^0 \subset e)$.
$\vdash < E,\ \star,\ e^0 >$ is a commutative monoid of idempotents.

A composed event is called a *process*.

Identity of events

Events, when considered as individuals, admit descriptions (duration, complexity, etc).

- D$_6$. $F = G \Leftrightarrow (\forall e)_E(Fe \wedge Re \Rightarrow Fe = Re)$.
- D$_7$. $R = S \Leftrightarrow (\forall e_1)_E(\forall e_2)_E ... (\forall e_n)_E(Re_1, ..., e_n \wedge Se_1, ..., e_n \Rightarrow Re_1, ..., e_n = Se_1, ..., e_n)$.

The identity criterion for events is given by

- P$_8$. $(\forall e_1)_E(\forall e_2)_E(e_1 = e_2 \Leftrightarrow \forall F : Fe_1 = Fe_2)$.

Leibniz's identity of indiscernibles

$$\vdash (\forall e)_E(e = e).$$

More relations between events

It is convenient now to define two important relations between events: *overlapping* and *separateness*. Two composed events (processes) overlap is and only if the have common events. Two events are separate if and only if they do not overlap. Formally,

- D_8. $e_1 O e_2 \Leftrightarrow (\exists e_i)_E (e_i \subset e1 \wedge e_i \subset e_2)$.
- D_9. $e_1 \backslash e_2 \equiv \neg(e_1 O e_2)$.

The World

The composition of all actual events is the World (W):

$$\neg(\exists e)_E \neg(e \subset W).$$

The World, W, should not be confused with the Universe, $\mathcal{U}$, the composition of all things in a thing-based ontology as the one given by Bunge (1977) and Romero (2013). The Universe can change, i.e. events and processes take place in the Universe. The World, the composition of all changes, can not change itself because it is not a thing. In an ontology of events, the totality of events is changeless, otherwise there would be a change not included in the totality, which is absurd. Events do not change, they *are* changes. In the sense used here for the words, the Universe can evolve, but not the World, which is fixed.

Order

Composition is not an ordering relation. We cannot adopt a simple relation of “before than", as Grunbaum (1973) did, because not all events can be ordered by such a relation without further specification. We need to introduce a stronger structure on the set of all events *E*, if we want to represent with this a set the World.

We stipulate that *E* is a metric space.

– D_{10}. E is a metric space if for any two elements e_1 and e_2 of E, there is a real number $d(e_1, e_2)$, called the *distance* between e_1 and e_2 in accordance with the postulates:

M1. $d(e_1, e_2) = 0$ iff $e_1 = e_2$.
M2. $d(e_1, e_2) + d(e_2, e_3) \geq d(e_1, e_3)$ with $e_3 \in E$.

$\vdash \; d(e_1,\, e_2) = d(e_2,\, e_1).$
$\vdash \; d(e_1,\, e_2) \geq 0.$

Only in case that $d(e_1,\, e_3) > 0$, there is a precedence relation between e_1 and e_3. I postulate:

- P_9. E is a metric space.

Then,

- D_{11}. The event e_1 *precedes* (or is *earlier than*) the event e_3 iff $(\exists e_2)_E[d(e_1,\, e_3) \geq d(e_1,\, e_2) + d(e_2,\, e_3)$.

In short, $e_1 \prec e_3$. Events such that $d > 0$, $d = 0$, and $d < 0$ are called *time-like*, *null*, and *space-like* events, respectively.

$\vdash \; <E, \prec>$ is a partially ordered set.

$\vdash \; (\forall e_1, e_2)_E \, [e_1 \prec e_2 \Rightarrow \neg(e_2 \prec e_1)].$

$\vdash \; \neg(\exists e)_E(e \prec e^0 \vee e^0 \prec e).$

- Reflexive: For all $x \in E$, $x \preceq x$.
- Antisymmetric: For all $x, \, y \in E$, $x \preceq y \preceq x$ implies $x = y$.
- Transitive: For all $x, \, y, \, z \in E$, $x \preceq y \preceq z$ implies $x \preceq z$.
- Locally finite: For all $x, \, z \in E$, Card $(\{y \in C | x \preceq y \preceq z\}) < \infty$.

Once the set of events has been equipped with a metric structure, I can make the fundamental semantic assumption of the event ontology: The World is represented by a metric space. In symbols:

$$- \ \mathrm{P}_{10}.\ E \triangleq W.$$

Here, E is a mathematical construct and W is the composition of all events, i.e. the maximal existent in an event ontology. It follows that

$$\vdash \quad \neg(\exists e)_E (e \prec W \vee W \prec e).$$

Things

To construct things out of events we introduce the operation of abstraction from a collection of individuals. Let us consider a formula with a single variable x that runs over events: \`(—— x ——)’. The formula predicates of each individual x such and such a description. We can abstract a virtual (i.e. fictitious) class from such a formula forming the collection and we call the class a “property”.

$$P = \{y : --y--\}.$$

Now, things can be constructed as classes of events sharing some properties, P, Q, etc:

$$X = \langle P,\ Q, ...\rangle\, e.$$

In this way things are bundles of events defined by shared properties, which are abstracted from conditions imposed on the events. The thing ‘Socrates’, for instance, is a cluster of events sharing their occurrence in Greece, previous to such and such other events, including events like ‘talking with Plato’, and so on.

Causality

Causality is a mode of event generation. Two events e_1 and e_2 are causally related iff there is at least a process p such that e_2 is component of p if e_1 is as well, and that is not the case if e_1 is not a component of p. We say then that e_1 is a cause of e_2.

$$e_1 \triangleright e_2$$

– P_{11}. There are events that belong to the same process but are not causally related.

Space-time

To go from ontology to physics we add more structure to the metric set that represents the totality of events.

P$^*_7$. Card $(E) = \aleph_1$.

P$_{12}$. The set E is a C^∞ differentiable, 4-dimensional, real pseudo-Riemannian manifold.

P$_{13}$. The metric structure of E is given by a tensor field of rank 2, g_{ab}, in such a way that the differential distance ds between two events is: $ds^2 = g_{ab}dx^a dx^b$.

P$_{14}$. The tangent space of E at any point is Minkowskian, i.e. its metric is given by a symmetric tensor η_{ab} of rank 2 and trace -2.

P$_{15}$. The metric of E is determined by a rank 2 tensor field T_{ab} though the Einstein field equations:

$$G_{ab} - g_{ab}\Lambda = \kappa T_{ab}. \tag{2}$$

$$\mathcal{ST} \hat{=} \langle E, g_{ab} \rangle \,.$$

Space-time is represented by an ordered pair whose elements are the manifold and the metric field.

A model of the World

$$M_{\mathrm{W}} = \langle E, g_{ab}, T_{ab} \rangle \,.$$

Since the ontic basis of the model is the *totality* of events, the World is ontologically determined. This does not imply that the World is necessarily *predictable* from the model. In fact, Cauchy horizons can appear in the manifold E for many prescriptions of T_{ab} (e.g. Joshi 1993). One thing is the World, and another our representations of the World.

Perspectives

- A quantum theory of space-time should start with P_7

$$- \ P_7. \ \text{Card}\ (E) < \aleph_0.$$

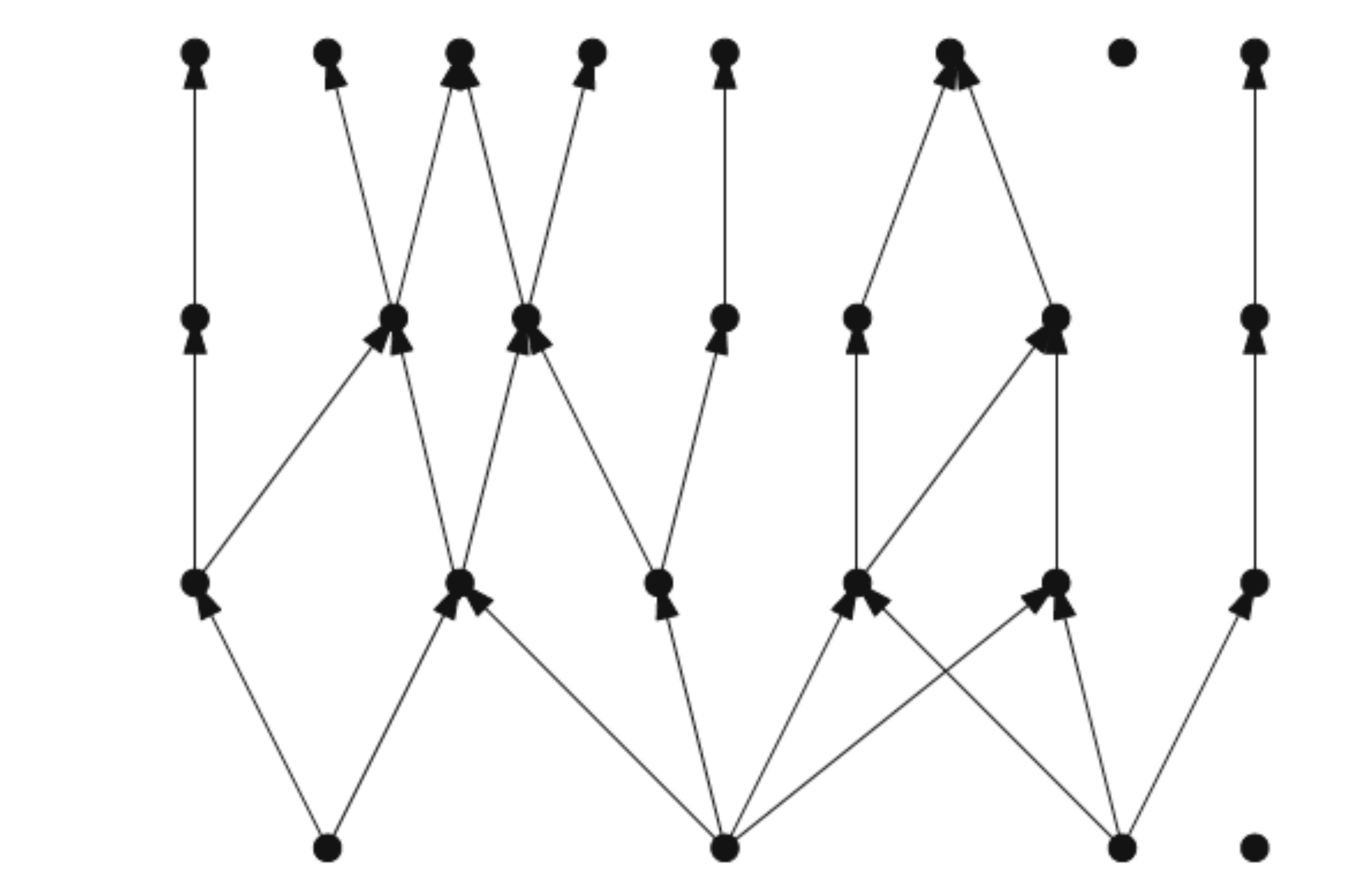
- Then, infinite magnitudes should not appear in the theory.

- Not all events are causally originated, but a poset of events might be adecuate to represent the most fundamental structure of space-time.

- Quantum gravity can be considered a theory about relations among basic events and the ontological emergence of space-time and gravity.

- It can be proved that the dimension, topology, differential structure, and metric of the manifold where a poset is embedded is determined by the poset structure (Malament 1977).

Thanks!



Group-like structures

	Totality*	Associativity	Identity	Divisibility	Commutativity
Magma	Yes	No	No	No	No
Semigroup	Yes	Yes	No	No	No
Monoid	Yes	Yes	Yes	No	No
Group	Yes	Yes	Yes	Yes	No
Abelian Group	Yes	Yes	Yes	Yes	Yes
Loop	Yes	No	Yes	Yes	No
Quasigroup	Yes	No	No	Yes	No
Groupoid	No	Yes	Yes	Yes	No
Category	No	Yes	Yes	No	No
Semicategory	No	Yes	No	No	No

*Closure, which is used in many sources to define group-like structures, is an equivalent axiom to totality, though defined differently.