

A PROGRAM FOR THE SEMANTICS OF SCIENCE

I. PROBLEM, METHOD AND GOAL

So far exact semantics has been successful only in relation to logic and mathematics. It has had little if anything to say about factual or empirical science. Indeed, no semantical theory supplies an exact and adequate elucidation and systematization of the intuitive notions of factual reference and factual representation, or of factual sense and partial truth of fact, which are peculiar to factual science and therefore central to its philosophy. The semantics of first order logic and the semantics of mathematics (i.e., model theory) do not handle those semantical notions, for they are not interested in external reference and in partial satisfaction. On the other hand factual science is not concerned with interpreting a theory in terms of another theory but in interpreting a theory by reference to things in the real world and their properties.

Surely there have been attempts to tackle the semantic peculiarities of factual science. However, the results are rather poor. We have either vigorous intuitions that remain half-baked and scattered, or rigorous formalisms that are irrelevant to real science. The failure to pass from intuition to theory suggests that semanticists have not dealt with genuine factual science but with some oversimplified images of it, such as the view that a scientific theory is just a special case of set theory, so that model theory accounts for factual meaning and for truth of fact. If we wish to do justice to the semantic peculiarities of factual science we must not attempt to force it into any preconceived Procrustean bed: we must proceed from within science. We should realize that a scientific theory is more than its mathematical formalism, and that this surplus is not describable in terms of 'operational definitions', let alone 'ostensive definitions'

My proposal is to explore and implement the following program for the semantics of science:

Problem: To investigate the semantic aspects of scientific theories.

Method: (i) To start by analyzing real specimens of scientific theory

with a view to disclosing its semantic components – mainly reference, representation, meaning, and degree of factual truth. (ii) To build exact (i.e., mathematical) theories about these semantic notions and their cognates. (iii) To check whether the explicata thus obtained are adequate, or at least relevant to live science.

Goal: To articulate the various special theories into a semantic theory of science capable of performing the following jobs. (i) To clarify and systematize the semantic aspects of scientific theories as distinct from the semantics of formal theories. (ii) To help scientists determine the precise reference and sense of their own theories – which reference and sense are often the object of heated debate.

This paper will outline the principles chosen to implement this program and will report briefly on some of the results obtained so far.

II. GUIDELINES

The semantical theories we wish to build should spell out the following principles:

(i) The symbols in a conceptual language designate constructs (concepts, propositions, or theories). In particular, a sentence is one among a number of signs designating a proposition. In short, we espouse conceptualism rather than literalism – not however a conceptualism of the Platonic variety.

(ii) Some of the constructs employed in science refer to real or supposedly real objects, such as protons, dinosaurs, and tribes. The set of putative referents of a factual construct may be called the latter's reference class. In other words, we adopt (critical) realism rather than either conventionalism or any form of subjectivism (e.g., operationism).

(iii) The reference class of a factual proposition and of its negate are the same. (No negative facts.) And the reference class of a truth functional compound of two or more propositions equals the union of the reference classes of the components.

(iv) Some factual constructs represent certain traits of their referents. For example, the atomic number of an atom represents the number of protons in its nucleus; and the matrix of the probabilities that the individuals in a community make transitions from one social layer to the other strata, represents the social mobility of the community. Such representa-

tions are literal not metaphoric, and symbolic rather than iconic. A factual theory, when formulated explicitly, should include statements indicating what the referents of its basic concepts are and what if anything they represent.

(v) Factual constructs have both an external reference and a sense. Two predicates representing different properties of one and the same thing differ in sense. For example, the concepts of electric conductivity and thermal conductivity are coreferential and even coextensive but not cointensive. Consequently we need a nonreferential theory of sense. Sense and reference are not mutually reducible: they must be treated on a par. They constitute the two components of meaning. Metaphor: regard R and S as the radius vector and polar angle, respectively, of a vector (meaning) in the plane of constructs. All the constructs with the same reference class (e.g., the various thermodynamic functions of a piece of matter) are represented by vectors with tips lying on a common circle. All the constructs with the same sense but different referents (e.g., the temperature values of different bodies) are collinear. A change in both reference and sense is represented by a pair of vectors which are neither collinear nor on a common circle.

(vi) The sense of a representing factual construct, such as a position coordinate or a mutation rate, is given by (a) its mathematical structure and (b) that which it represents. In an axiomatized factual theory both aspects should be taken care of (not just the formal aspects). And in such a theory it is the axioms in which the construct occurs that ultimately determine its sense – or, as we may say, such axioms determine the gist of the construct.

(vii) Sense is contextual: strictly speaking there are no categorematic terms. While extrasystematic sentences are hardly significant, the sense of a construct belonging to a theory is assigned by a good portion of the whole theory – in fact by all the constructs that are logically related to the given construct. Change the theory and ‘the same’ construct (or rather the same symbol) is likely to change its sense, even though its reference may remain invariant. Therefore the concept of sense should be relativized to a theory.

(viii) In a scientific theory sense and reference are either assumed or derived. The search for sense must therefore proceed both upwards, to the basic assumptions, and downwards, to their logical consequences. The

former will constitute the gist, the latter the content of the construct. A deductively isolated predicate, if there were any, would have no precise sense at all.

(ix) Conjunction enriches. Therefore the sense of a conjunction should include the senses of the conjuncts. The sense of a negation should equal the complement of the original sense in the given context. And if two constructs are identical so must be their senses. These assumptions should suffice as a foundation for a theory of sense.

(x) The 'inverse law' of intension and extension should hold when formulated in this way: "If the sense of *A* is included in the sense of *B*, then the extension of *A* includes (or is equal to) the extension of *B*". This formula should be a theorem in the theory of sense.

(xi) Any talk of meaning variance or invariance should be accompanied by a theory of meaning – otherwise it will be just loose talk. The 'amount' of change in the meaning (sense *cum* reference) of a construct when adopted by a new theory should be expressible in exact (e.g., set theoretic) terms.

(xii) From a semantic standpoint a factual theory is an interpretation of a mathematical formalism. One and the same formalism may be assigned alternative factual interpretations, each of which gives rise to a different factual theory. A factual interpretation is, roughly, an assignment of factual meaning. In other words, a construct in factual science is a mathematical construct together with a factual interpretation.

(xiii) Whereas interpretation bears on exact concepts, elucidation bears on inexact ones. Interpretation is, roughly, the converse of elucidation. Thus probability elucidates or exactifies the concept of possibility and, conversely, possibility interprets probability. (Incidentally, for this reason, i.e., because there exists a quantitative calculus of possibility, science makes no use of modal logics.)

(xiv) Meaning is prior to truth – *pace* Frege and the Vienna Circle. Change the interpretation of a formula and its truth value may change. Moreover, most formulas are never tested for truth, hence are never assigned a truth value: they have to wait in a semantic limbo. And yet they are supposed to satisfy the laws of logic and to have a definite meaning. And such a meaning must be understood before any experiment can be designed to find out truth values.

(xv) Truth conditions, important as they are in elementary logic, become blurred in science. To begin with, a truth condition for a factual

sentence cannot possibly determine the significance of the latter. (While every statement comes with a more or less definite meaning, it may not have been assigned a truth value. And acquiring one won't change its sense and reference.) Factual truth conditions are the business of scientists and methodologists, not of semanticists. And, rather than clear cut biconditionals ("A is true iff B"), in actual practice a truth condition consists of an ill articulated set of necessary conditions for high, medium, or low degree of factual truth. Moreover, the assignments of factual truth values are provisional.

(xvi) Truth, a semantic property, is a property of propositions not of physical objects such as written or spoken sentences. And factual truth is a property of statements with a factual reference. But complete truth is not easy to come by in factual science: the best we get is approximate truth. Moreover we always get relative truth, i.e., truth degree gauged against some proposition taken to be true. For example, let F and G be two functional statements representing, each in its own theory, a given feature of a supposedly real thing, such as the velocity of a body falling freely in the vacuum. In particular, suppose that

$$\begin{aligned} F &= \lceil v = v_0 \rceil && \text{(Aristotle)} \\ G &= \lceil v = v_0 + gt \rceil && \text{(Galilei)}. \end{aligned}$$

A possible formula for the truth value of F given G (assuming G to be true) is the absolute value of the ratio of the values of the functions for the same thing:

$$V(F | G) = \frac{v_0}{v_0 + gt}.$$

This relative degree of truth approaches 0 (is near unity) for large (very small) values of g or of t . Alternative formulas are possible.

This being the case it behooves the semanticist to elucidate this concept of relative and approximate truth and, in general, the concept of degree of truth. Since this concept of partial factual truth is important, it is unlikely to be definable. The best strategy may be to make it into a primitive concept of a special theory.

(xvii) In attempting to build a theory of partial and factual truth we must resist the temptation to equate it with probability or some function of probability. The main reason for this is that there seems to be no

procedure for assigning probabilities to propositions other than by arbitrary fiat. We must also resist the temptation to resort to many-valued logics. The main reason for this is that mathematics, the skeleton of factual science, has ordinary logic built into it. A theory of partial and relative truth of fact should then presuppose ordinary logic. To this end, the logical truth values may be regarded as just the unit and the zero elements of the Boolean algebra of (formally equivalent) statements. This structure can then be imposed any number of alternative metrics. Any member of the unit element of the set (i.e. Frege's *das Wahre*) can be assigned the real number 1, and any member of the zero element of the set (*das Falsche*) the real number 0.

(xviii) Desiderata for a theory of partial truth consistent with ordinary logic: (a) Truth is a (partial) function from the set of propositions into an interval of the real line, e.g., $[0, 1]$; i.e., $V(p) = v \in [0, 1]$. (b) $V(\sim p) = 1 - V(p)$. (c) If the propositions p and q are logically independent (not interdeducible), then $V(p \& q) = V(p) \cdot V(q)$.

(xix) The notion of extension, though important, is derivative, for it depends on the concepts of reference and of truth. The strict extension of a concept is the collection of those of its referents that happen to have the property represented by the concept. The lax extension of a concept (whether exact or vague) is the set of referents that satisfy it approximately or to a given extent. (Incidentally, do not mistake 'extensional' for 'truth-functional' and 'intensional' for 'non-truth functional', as *PM* did. Science is not purely extensional, as every one of its constructs comes with an intension. But science employs only ordinary (truth functional) logic. If for no other reason the semantics of science has no use for what are often, mistakenly, called "intensional [non truth functional] logics".)

(xx) Lastly, a piece of methodological advice: In expanding the preceding principles into theories, try and keep them together. Do not attempt to handle each semantic concept in isolation but try to articulate the theories of the various concepts (reference, representation, sense, truth, extension, etc.). The reason is plain: these concepts *are* inter-related.

III. PREVIEW OF RESULTS

Here go, in quick succession, some inklings of the results obtained so far in implementing the program formulated in the previous section.

(i) *Designation* is construed as a certain many-one function from signs to constructs.

(ii) *Reference* is elucidated as a certain function from constructs to things. More exactly, two reference functions are introduced, one from predicates to sets of individuals, the other from propositions to sets of individuals. And the factual reference functions are the restrictions of the preceding functions to sets of factual items.

(iii) *Denotation* is defined as the composition of designation and reference.

(iv) *Representation* is clarified as a certain relation from constructs to aspects of things. Whatever represents refers but not conversely.

(v) Two representing constructs in a theory constitute *equivalent representations* of the same factual item if they can be freely substituted for one another in every basic law statement of the theory.

(vi) The *purport* or upward sense of a construct x in a set C of constructs, closed under the logical operations, is the principal ideal generated by x in C . In other words, the purport of a construct is the collection of its logical forebears.

(vii) The *gist* or essential sense of a construct is a subset of its purport. In an axiomatic theory the gist of a construct is the set of axioms in which the construct occurs. Whence the semantic import of axiomatics.

(viii) The *import* or downward sense (or content) of a construct x in a context C closed under the logical operations is the principal filter generated by x in C . That is, the import of a construct is the totality of its logical progeny.

(ix) The *full sense* of a construct is the union of its principal ideal and its principal filter, i.e., of its purport and import.

(x) If a construct has no place in a deductive system, or if its place is ignored, we assign it a horizontal sense or *intension* (or comprehension). If p and q are constructs of the same type (either predicates or propositions) and if they can be conjoined, then (a) $I(p \& q) = I(p) \cup I(q)$; (b) $I(\neg p) = \overline{I(p)}$; (c) if $p = q$ then $I(p) = I(q)$ but not conversely.

(xi) Consider a Boolean algebra of either predicates or propositions. Then the family of their intensions is a ring I of sets: the ring of intensions. Define in I the function $\delta: I^2 \rightarrow I$ with $\delta(p, q) =$ The symmetric difference between the intension of p and the intension of q . Then δ defines neighborhoods in a topological space. And a neighborhood of p in this space

is constituted by the conceptual relatives of p . This elucidates Wittgenstein's vague notion of *family resemblance*.

(xii) *Meaning* is taken to be a property of constructs. The meaning of x in C is the ordered pair: \langle Sense of x in C , reference class of x in C \rangle . Consequently two constructs have the same meaning just in case they have both the same sense and the same referents, i.e., if 'they' are the same construct.

(xiii) *Signification* is regarded as a property of signs. It may be construed as the composition of designation and meaning. The significance of a sign is the meaning of the construct the sign designates. Signs may thus have a vicarious meaning (sense *cum* reference). A sign is nonsignificant just in case it designates no meaningful construct.

(xiv) *Synonymy* (in a language) is equal significance, hence identical meaning (same sense and reference) of the underlying constructs. We can do better than just defining synonymy: we can compare symbols as to significance, since our theory of significance rests on a theory of meaning couched in set theoretic terms.

(xv) *Truth* is construed as a real function V on a subset S_D of the set S of statements. (Being a partial function on S , V makes room for truth value gaps, which are only too conspicuous in science.) Because the family of equivalence classes of the propositions in S_D has a Boolean structure, the whole thing becomes a metric Boolean algebra.

(xvi) The desiderata xviii imposed on V in Section II determine a function V that looks adequate, in the sense that it seems to be consonant with actual patterns of scientific inference. Moreover, actual statements in theoretical science can in principle be assigned definite *degrees of truth* (but not probabilities). Thus if a statement p has been found by experiment to be in error by the amount ε , we set $V(p) = 1 - \varepsilon$, and we deduce $V(\sim p) = \varepsilon$. In this way the theory of truth can be conjoined with the theory of scientific inference (a branch of mathematical statistics, not of inductive logic).

(xvii) The quantitative concept of truth allows us to define certain qualitative concepts such as the one of set of confirmers (or else of infirmers) of a given statement. In fact, consider the (quasi) distance function $d: S_D \times S_D \rightarrow [0, 1]$ such that $d(p, q) = |V(p) - V(q)|$ for any p and q in S_D . An open neighborhood of a point p (the family of its alethic relatives) is the *set of confirmers* of p .

(xviii) *Definite description* loses much of its glamour in our semantics, for it is construed as indicating uniqueness rather than both uniqueness and existence. Moreover, in one of our construals 'The length of *a*' is just the first half of the complete functional statement 'The length of *a* equals *b*'.

(xix) *Analyticity*, central to the semantics of logic and mathematics, is rather unimportant in the semantics of science provided it is conceived in a narrow way. The construal I propose is this: A formula is analytic in a given theory iff it holds under all interpretations (in all models) of the theory or is a definition in the theory. The great divide is not analytic/synthetic but formal/factual. And the analytic formulas constitute a smallish (though infinite) subset of the set of formal formulas.

(xx) The upshot of our investigation will be a body of theories that may be regarded as included in, or at least tangential to, epistemology. Indeed, they are concerned with constructs belonging to the body of our conjectures about the world. The relation of this semantical theory to metaphysics is but slight. We distinguish three concepts of existence: neutral [paradigm: $(\exists x) Px$], conceptual [paradigm: $(\exists x) (Px \ \& \ x \text{ is a construct})$], and physical (paradigm: $(\exists x) (Px \ \& \ x \text{ is a physical object})$). Existential quantification, unless qualified, is ontologically neutral. Logic and mathematics have nothing to do with ontology except that they should be

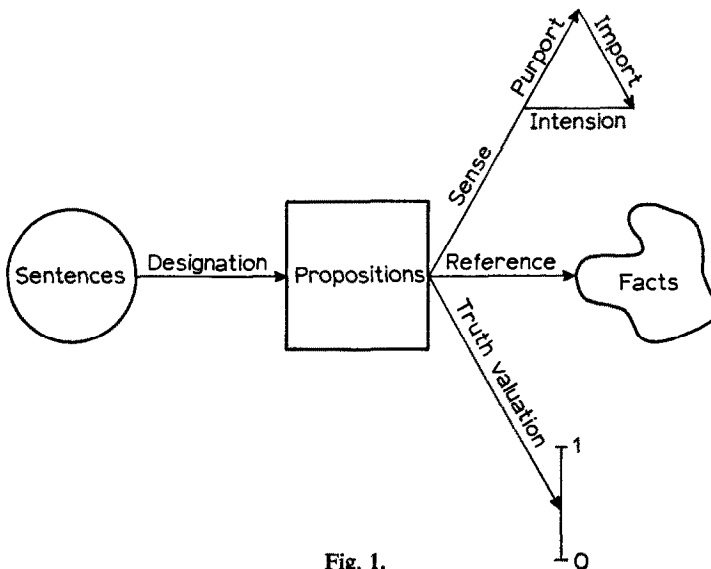


Fig. 1.

respected by the latter. Only scientific theories make 'ontological commitments', or rather assumptions. And they need an ontologically neutral logic and mathematics.

Figure 1, restricted to statements, displays the architecture of our semantic theory.

IV. AN APPLICATION

Imagine a theory of drives or urges, such as hunger, that assumes the intensity D of every drive to be a certain function of some physiological misalignment or imbalance i . More precisely, assume D to be nil below a certain threshold i_0 and to have a sigmoid shape above i_0 . One of the infinitely many functions that will comply with this loose description is the function D from reals into reals such that

$$D(i) = au(i) i^2 / (b + i^2) \quad H$$

where a and b are positive real numbers, and $u(i) = 1$ for $i > i_0$ and 0 for $i \leq i_0$. The above is just the central hypothesis of our bogus theory. The remaining fundamental statements in the theory spell out exactly what the various symbols designate, what the corresponding constructs are about, what if anything they represent, and what their dimensions and units are. For example, there will be a statement to the effect that the domain of the independent variable i is a certain class M of organisms, say humans, while the range of i is the positive real line. On the other hand the theory will contain no indication concerning its own test. In particular it will not contain hypotheses serving to objectify and measure the drive intensities $D(i)$. In principle there are several such objectifiers or indices, either physiological or behavioral, hence several possible techniques of measurement. Usually it is up to the ingenuity of the experimenter to conjecture, test, and use any such relations between covert qualities and their manifestations. In any event, the objectifiers and their measurement are relevant to the test for the truth of the theory, not to its meaning.

A cursory semantic analysis of the quasitheory sketched in the preceding lines yields the following results.

Reference class of i = Reference class of $D = M$ (mankind)

Factual interpretation of $i = i$ represents a physiological imbalance of a certain type (e.g., deficiency of sugar in blood).

Factual interpretation of $D = D(i)$ represents a drive or urge of a certain

type (e.g., hunger) as felt by an organism of the kind M suffering imbalance i .

Sense of i = The set of physiological formulas in which i occurs.

Sense of D = The set of formulas entailing or entailed by H and its companions.

Gist of i = The basic (postulated) formulas among all those containing i .

Gist of D = $\{H, \text{The above factual interpretation of } D\}$.

So much for the *postulated* sense and reference. Now for the *derived* meanings. They are inferred from the preceding plus an analysis of the mathematical roles the constructs concerned play in the central hypothesis H .

Reference class of a = *Reference class of b* = M .

Factual interpretation of a = Maximum drive strength.

Factual interpretation of $(1/b)$ = Strength of the curbing (inhibition) of further drive increases.

Once meanings have been assigned we may proceed to find out truth values on the basis of some body of empirical evidence E relevant to the central hypothesis H . Thus we may pronounce H almost true if $V(H | E) = v$ comes close to unity. Whether a statement such as the preceding one is to be called a *truth condition*, is a matter of taste. In any case it is a far cry from a truth condition in elementary logic. And it contributes nothing to the meaning of the theory.

V. CONCLUDING REMARKS

Our program is ambitious, as is any attempt to match life (in our case real science) with virtue (e.g., exactness). We want our semantics to be not only *simia mathematicae* but also *ancilla scientiae*: built *more geometrico* and at the same time relevant, nay useful, to live science. The goal of exactness may sound arrogant but is actually modest, for the more we rigorize the more we are forced to leave out of consideration, at least for the time being. As to the service intention: we should try to be of some help to science because the latter faces semantic problems but has no tools of its own for solving them. If it had such tools scientists would not engage in spirited polemics over matters of sense and reference, as they often do. Witness the debates on whether the relativistic and quantum theories are concerned with sentient observers, whether population genetics

refers to populations taken as wholes, whether psychology is actually concerned with the brain, and whether the sense of a theory is excreted by its mathematical formalism or is determined by the way the theory is tested.

A semantics of science should help settle these and similar issues. Moreover it should give sound advice as to how to formulate scientific theories so as to avoid such imprecisions and ambiguities as may give rise to debates of the kind. Constructing such a semantics, both exact and relevant to science, should be more rewarding than either manufacturing neat but irrelevant theories or pursuing erratic polemics on meaning and meaning changes.*

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