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BUCKETS OF WATER AND WAVES OF SPACE: WHY SPACETIME IS PROBABLY A SUBSTANCE*

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This paper sketches a taxonomy of forms of substantivalism and relationism concerning space and time, and of the traditional arguments for these positions. Several natural sorts of relationism are able to account for Newton's bucket experiment. Conversely, appropriately constructed substantivalism can survive Leibniz's critique, a fact which has been obscured by the conflation of two of Leibniz's arguments. The form of relationism appropriate to the Special Theory of Relativity is also able to evade the problems raised by Field. I survey the effect of the General Theory of Relativity and of plenism on these considerations.

1. Introduction. For those philosophers who labor in the vineyards of ontology, space and time perennially remain among the most challenging and annoying of species. To those who regard them as weeds to be eradicated they prove a hardy and resilient adversary. Attempts to extirpate them often result in the destruction of all other vegetation, so intimately entwined are their roots among the various varieties of more substantial produce. Yet those who seek to cultivate them encounter instead a delicate and sickly flora, remaining ever wan and wilted. One cannot but admire Kant's response to this dilemma: Unable either to cast them out or to admit them as organic, he styled them instead trellises of human manufacture, rigid frames essential to the sustenance of the foliage. But leaving such creative husbandry aside, granting that the world *an sich* is

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a spatiotemporal object, we must face a fundamental problem: Are space and time entities in their own right?

In the traditional nomenclature, approaches to this question fall into two groups. *Substantivalists* regard space and time (or spacetime) as capable of existence independent of material objects. According to some theories substantival spacetime serves as the subject of which other properties are predicated. For others it serves rather as a container which is populated by material objects. *Relationists*, in contrast, seek to construe space and time as hypostatizations, unhealthy excreta of a diseased language. Spatiotemporal relations between objects may be real enough, but to infer that space and time are entities capable of independent existence is to commit the fallacy of misplaced concreteness. In these terms the battle was joined between Newton and Leibniz, and in these terms it continues to this day.

More than nomenclature has been inherited. All too often debates over the status of space and time have continued in strict conformity with the plan set by Newton and Leibniz. Newton taxed the relationist with the problem of inertial effects. Mere change in relative position of bodies is not systematically correlated with observable dynamical phenomena, hence absolute space and time must be introduced to account for observable absolute acceleration. Mach provided the sketch for a relationist response to Newton: Inertia is caused by relations to distant masses. The question naturally arises whether the General Theory of Relativity (GTR) complies with the Machian requirement that inertial structure be fixed by relations to material bodies. Since the GTR fails this test, the substantivalist still has the upper hand (see Sklar 1976). On the other side, Leibniz's main assault turns on the metaphysical distastefulness (or absurdity) of ontologically distinct but observationally indistinguishable states of affairs. Newtonian absolute space seems to allow the possibility of such states since the whole material world could be displaced or set in motion in space without any consequent change in appearance. The most recent progeny of this line of attack is the claim that a version of Leibniz's argument formulated in the context of the GTR leads substantivalism into direct conflict with determinism (Earman and Norton 1987; Earman 1989, chap. 10).

Unfortunately, physics and physical phenomena do not bear on this issue in the way the sketch above suggests, and many of the old considerations in favor of one or the other side no longer carry any weight. These mistaken projections of the Leibniz/Newton controversy into the contemporary scene arise primarily from two sources. First, two of Leibniz's arguments are easily conflated, obscuring the structure of each. Each argument originates with observations about *how the world might have been* if space were absolute. The first notes that the world might have been situated elsewhere in absolute space, the second that it might have had a different absolute velocity. Each of these possibilities is then said to engender an unacceptable state of affairs. But the two arguments operate by means of entirely different principles, and turn on different considerations. In particular, only the second possibility gives rise to the existence of empirically undeterminable matters of fact.

The second problem arises from focusing too narrowly on the particular relations Leibniz used in his account. Relationism about spacetime structure takes many forms, depending on the sorts of relations postulated. The form of relationism most congenial to special relativistic theories, for example, is not the form Leibniz considered, and does not fall prey to many of the objections raised against him. We need a taxonomy of relationisms, and a careful consideration of their strengths and weaknesses.

Before we begin, one neighboring issue must be noted and carefully distinguished. The problem of the ontological status of space and time is closely connected to puzzles about the vacuum. The connection is multifarious and oblique. Descartes, for example, subscribed to a principle which would force the relationist to postulate a plenum, namely, that for two objects to be at a distance from one another there must be an extended object between them (see [1644] 1984, part 2, principles 17 and 18). Points *A* and *B* can be four meters apart only if an object four meters long is interposed. The relationist has no reason to subscribe to this doctrine, or to deny the acceptability of talk of vacua. If the distance relation between objects is logically primitive and independent of the extension of intervening objects, then "vacuum" can easily be paraphrased away. To say a vacuum exists midway between *A* and *B* is just to say that nothing at all exists which has the distance relations of being two meters from each of them. Such a relationist is no more ontologically committed to the vacuum as an entity than one who accepts that there is nothing in the fridge is ontologically committed to the existence of nothing.

So the relationist need not posit a plenum. But the introduction of a plenum does significantly alter the bearing of several of the arguments that we will consider. To better highlight this, in the first four sections I will treat only a particle ontology which allows a vacuum. Only in the last sections will a field ontology, which demands a plenum, be considered.

Section 2 is a brief examination of Newton's bucket argument and some relationist responses. Section 3 anatomizes Leibniz's arguments against substantivalism, sorting out two distinct strands of reasoning that are often confused and commingled. The next section reviews some new considerations offered by Hartry Field. The fifth examines how the Special Theory of Relativity (STR) affects the analyses previously offered. Section 6 adds both the General Theory and the field ontology to the brew.

2. The Bucket. In the Scholium to definition 8 of the *Principia* ([1729] 1962) Newton described the two experiments—one actual and one hypothetical—which would inject physical considerations into the debate over space and time. The actual experiment demonstrates that the water in a bucket sometimes endeavors to recede from the central axis. It does so, we say, when the water rotates. But the dynamically effective rotation is not the rotation of the water relative to the bucket, as Newton observes. Newton concludes that the dynamical effect is caused by motion relative to absolute space.

The thought experiment is meant to drive this point home. In an otherwise empty world, two globes yoked by a cord could be in any number of different physical states. If the globes revolve about their common center of gravity there will be a tension in the cord. Further, experimentation on the system would reveal that forces applied to one set of faces of the globes increase the tension while forces applied to the opposite faces reduce it. Newton claims we could thereby determine both the rate and the sense of the revolution of the globes even if there existed no reference body relative to which they were spinning. So the rotation must be relative to absolute space.

Why should this thought experiment, assuming we accept its validity, cause problems for the relationist? Suppose one believes that the only spatiotemporal facts are facts about the temporal duration between events and about the spatial distance between material bodies *at a given time*. Then the situations with different observed tensions exhibit exactly the same spatiotemporal structure since the relative distances of the globes to one another and to bits of the cord are always the same. So a relationist of this ilk lacks the explanatory resources to account for the variation of tension in the cord. According to the relationist the various situations—with what the substantivalist describes as various degrees of rotation—are all absolutely identical in structure. All of the physical facts are the same, so no cause for the variation of the tension could be found.

How does the introduction of absolute space provide Newton with the wherewithal to explain the dynamical effects? Since the parts of absolute space endure through time, they provide a reference frame by which distance relations can be defined not only between bodies at a time but also between bodies at different times. Even a single body either remains in the same absolute place, and hence is at absolute rest, or is not. In terms of a four-dimensional spacetime, the relationist described above posits a distance relation which takes as its domain only pairs of simultaneous material events, while the substantivalist, by use of the world-lines of parts of absolute space, extends the domain to all pairs of events.

As should be clear, Newton's ability to account for inertial effects does not immediately depend on the postulation of space as a substance. Substantival absolute space serves only as a means of extending the domain of the distance relation to include pairs of nonsimultaneous events. The bucket argument, then, is not effective against relationism per se but only against a certain brand of that doctrine. Let us call the relationist position which posits, in addition to the usual temporal relation, only distance relations between simultaneous events *Leibnizian relationism*. If one accepts the validity of Newton's thought experiment (which Mach would contest), Leibnizian relationism has been demonstrated to be too ontologically poor to explain certain physical phenomena.

To defeat one tribe of relationists, though, is not to vanquish the whole nation. The most direct way for a relationist to overcome Newton's argument is simple: Accept his ontology of *relations* while rejecting substantival absolute space as its supporting framework. Such a *Newtonian relationist* could still maintain that all spatiotemporal facts are facts about the relations between material bodies or, speaking four-dimensionally, material events. But for the Newtonian relationist those relations include a distance relation between noncontemporaneous events.

The Newtonian relationist inherits almost all the power to make ontological distinctions that the Newtonian substantivalist has. For example, the Newtonian relationist can accommodate absolute motion and rest. A body is at absolute rest if the distance between its successive temporal stages is zero. The revolving spheres instantiate a different set of distance relations than do the resting spheres, and different rates of rotation can be distinguished. The revolution of the spheres *is* a relative phenomenon: not of the spheres relative to one another but of the later temporal stages of the system to its earlier stages. Inertial forces do not disbar relationism, but they do place requirements on the nature of the relations posited.

Newtonian relationism is not a doctrine that has been defended by any historical figure. The possibility of the position does not become evident until one adopts a four-dimensional picture and an event ontology. Even if it has not been championed, Newtonian relationism is worthy of our notice. It is not just Newtonian substantivalism fitted out in a misleading vocabulary; as we will see presently, Newtonian relationism would avoid one of Leibniz's objections against Newtonian substantivalism. It demonstrates that Newton's bucket argument is not effective against relationism per se. Relations that are sufficiently rich can provide the means of explaining inertial effects. This lesson will become germane when we turn to the Special Theory, where the only natural relationism is one which would escape Newton's criticism in just this way.

3. The Relationist Counterattack. Leibniz assailed Newtonian absolute space and time primarily by two related arguments, each involving a so-called *Leibniz shift*. These arguments turn on the observation that if space and time are independently existing entities within which material bodies are placed then one must accept the metaphysical possibility of ontologically distinct but observationally indistinguishable states of affairs. Such a possibility is then said to lead to conflicts with either the Principle of Sufficient Reason (PSR) or the Principle of Identity of Indiscernibles (PII).

That the shift operations central to these arguments have come to be called after Leibniz is ironic, for it was Clarke who presented each as an argument *against* relationism. According to Clarke, the shifts would create situations which are evidently ontologically distinct but in which all spatiotemporal relations between bodies remain unchanged (Leibniz 1956, 20–21, 32). Leibniz, in a classic modus ponens/modus tollens reversal, argues that since only those relations can be observed, the supposed shifts create no real change at all.

Leibniz's use of the shifts is certain to strike the modern reader as less than compelling since neither the PSR nor the PII enjoys at present unquestionable philosophical credentials. A complete appreciation of Leibniz's arguments, though, still requires a close examination of their structure. The arguments employ principles that are obscure and can mislead. Evidence of this comes from the fact that the two shift arguments are often taken to have the same structure although they actually depend almost entirely on different considerations. Let us sharpen our logical scalpels and dissect these relationist ploys.

The *static Leibniz shift* involves considering the possibility that the entire material universe may have existed displaced either spatially or temporally from its actual location in absolute space and time. If space and time are independently existing receptacles of objects, then the material world might have been situated three meters to the north of its present location, or might have been begun three centuries earlier, or "placed the quite contrary way, for instance, by changing East into West" (ibid., 26), without any consequent change in the relations between bodies.[1] According to the substantivalist such static shifts do describe possible states of the universe distinct from its actual state.

The *kinematic Leibniz shift* proceeds instead from the observation that according to Newtonian substantivalism the material world as a whole has an absolute velocity, a state of motion relative to absolute space. But

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[1] The substantivalist need not admit that *these very bodies* could have been relocated, but would concede that bodies qualitatively identical to these might have been so placed. For simplicity, I will speak of the same bodies as being moved until the question of the identity of the bodies becomes relevant.

since only absolute acceleration has any dynamical consequences, uniform absolute motion would produce no observable change. Just as in the static case, we can imagine a plethora of possible states of the world, each with a different absolute velocity, which are distinct substantivalist models of Newtonian dynamics but in which the (Leibnizian) spatiotemporal relations between objects are identical.

In each case the substantivalist must admit the existence of a multitude of distinct physical possibilities where the relationist admits only one. Why should this admission trouble the substantivalist? Two lines of reasoning can be pursued. First, since the various supposed states exhibit the same relations between bodies, if those relations are the only observable spatiotemporal facts, the states would be empirically indistinguishable from one another. Hence by the PII they would be the same state. Second, if the states really represent distinct ontological possibilities and absolute space and time are completely homogeneous and isotropic, God could have no grounds to prefer creating one rather than another. The existence of the universe would therefore constitute a violation of the PSR. So either the PII or the PSR (along with the existence of the world) refute the substantivalist view.

In short, both the static and kinematic shifts, if real possibilities, would result in *ontologically distinct but observationally indistinguishable states of affairs*, and these are supposed to be metaphysically objectionable.

As sketched above, the two arguments display fundamentally the same structure. This is, however, an illusion engendered by imprecision concerning the notion of *observationally indistinguishable* states of affairs. Let us cut a bit deeper.

It is tempting to explicate the idea of observationally indistinguishable states of affairs by the following fanciful scenario. Imagine God with various possible dispositions of the material universe before Her mind. Now suppose God chloroforms you and you awaken to find yourself in one of these possible universes. No amount of observation will permit you to determine which possibility She actualized. You could not tell where She located the material world or what absolute velocity She imparted it even if you could observe all observable phenomena to the end of time.

This sort of fanciful picture encourages the equation of the static and kinematic shift arguments. But when we try to eliminate the fanciful element a different result emerges.

Consider first the kinematic shift. It forces the substantivalist to admit that all observable phenomena are consistent with various possible states of absolute motion of the entire material world. The universe as a whole may be at rest, or travelling uniformly five meters per second due north, or 888 meters per second in the direction between Earth and Betelgeuse, and so on. According to Newtonian dynamics *no possible observation* can reveal its actual state of motion. So Newton must postulate that the universe has a physically real but empirically inaccessible property. In this sense, states which differ only in their net absolute velocity are observationally indistinguishable.

What if we try to make the same argument using the *static* shift? Various positional states of the universe as a whole are possible: It could be created so my desk is *here,* or three meters north of here, or 888 meters from here in the direction from Earth to Betelgeuse, and so on. Which is the *actual* state of the world? Now the answer is easy: In its actual state, my desk is here, not three meters north or anywhere else. So in the kinematic case, unlike the static case, sensible physical questions can be asked but cannot be answered by observations. To even formulate the appropriate question in the static case one must indexically pick out a spatiotemporal location, and it is then no great trick to observe what material body that location actually contains.

A universe created 15 billion years ago is observationally distinguishable from one just like it (i.e., having a qualitatively identical total history) which began within the last four minutes. Things would look *awfully* different if the big bang had occurred in the last half hour. Of course, if the big bang had occurred four minutes ago then in another 15 billion years there might be someone who looks just like me writing a sentence that looks just like this. But that person would have no difficulty determining that he is not alive *now,* just as I have no difficulty knowing that I will not be alive *then*. And though he would produce the same characters and phonemes as I, the indexicals in his language would guarantee that his utterances would not mean the same thing as mine.

The essential difference between the static and kinematic cases lies in the semantical role that indexicals or demonstratives play for terms denoting *places* as opposed to the roles they play in terms denoting *velocities*. Since all absolute places are qualitatively identical, the only way we can possibly refer to them is either by direct ostension or by using a definite description which makes reference to some material object (e.g., "the absolute place occupied by the largest pulsar in 1963"). Either method allows us in principle to identify the absolute places and so to determine what objects actually occupy them (at some given time). We can then formulate meaningful counterfactuals about worlds where everything would be displaced from its actual location, but we can also be assured that they *are* counterfactuals, that they do not describe the world as it is.

In contrast, absolute velocities can be described without reference (either indexical or descriptive) to the velocity of any material object. Newton describes clearly the state of absolute rest without identifying any observable object which is at absolute rest. We can specify an absolute velocity by identifying a direction and a rate, making no reference to the velocity of any material object. We can then sensibly ask about the Earth's absolute velocity without having, even in principle, a means to determine what it is. But we can only sensibly ask about the position of the Earth by asking for its position relative to some determinate set of coordinates, and the linguistic wherewithal needed to establish the coordinates also provides us the means of answering the question. In sum, the *only* way that the static shift can be formulated is something like, "what if God had created the material universe oppositely oriented *to the way it is oriented now?*", and this is clearly a counterfactual situation. But we can ask "what if God created the material universe at absolute rest?", not knowing whether we describe a counterfactual situation or not.

For the substantivalist, terms such as "here" or "now" can be used to drive linguistic pegs into the fabric of absolute space and time. Without such pegs, the static Leibniz shift cannot even be formulated. The relationist will no doubt contest this interpretation of the indexicals: If no substantival spacetime points are available to pick out, such terminology must be explicated instead in terms of relations to particular physical bodies. But to object this to the substantivalist is a manifest petitio principii: If such points do exist, there is no reason that we cannot directly refer to them.

If one expunges fantasies about being kidnapped by God, one finds that the static shift does not result in an indistinguishable state of affairs, nor does it imply that there are any real but empirically undeterminable spatiotemporal facts about the world. The world described by the shift may be *qualitatively* indistinguishable from the actual world in the sense that no purely qualitative predicate is true of the one which is false of the other. But we have more than purely qualitative vocabulary to describe the actual world; we have, for example, the indexicals, without which the Leibniz shift cannot be described. So in the context of the static shift, the PII is of no use.

The PSR problem remains, for God could have had no rational grounds to create the universe in one neighborhood of absolute space or time rather than another. But eliminating God from this scenario leaves no problem at all; since no such choice was ever made, there need never have been rational grounds to make it.

As only the PSR and not the PII is relevant in the static case, so only the PII and not the PSR is relevant in the kinematic one. If God had a choice in setting the net absolute velocity of the material world, then the PSR, far from making a decision impossible, would have constrained Her to one unique choice. For if She set it in motion, She would have to choose some direction. But the isotropy of absolute space prohibits any grounds for preferring one direction over another. The unique absolute velocity compatible with the isotropy of space, the one, so to say, isotropic velocity, is zero. If God created the material universe, She created it at rest.

The possibility of a kinematic shift would therefore create no difficulty for the Creator. But having dispensed with God, we cannot provide theological grounds to prove that the absolute velocity of the universe is zero. The atheistic Newtonian must admit that there are empirically inaccessible facts, that *in this sense* our world considered as at rest and as having any absolute velocity are indistinguishable. So here the PII may have some bite.

The Newtonian can, of course, simply deny the PII as a metaphysical principle. Man is not the measure of all things, and there is no reason to believe that all real properties must fall within the power of human observation. Still, one should be made at least uncomfortable by the postulation of empirically inaccessible physical facts. Ceteris paribus, one would prefer a theory without them.

In this case the Newtonian can adopt a theoretical change that eliminates the discomfort, rejecting Newtonian absolute space and time in favor of substantival neo-Newtonian spacetime. Such a spacetime has exactly the structure that Newtonian dynamics requires: full spatial metrical structure only on the simultaneity hypersurfaces but an affine connection throughout. In neo-Newtonian spacetime absolute acceleration is well defined but absolute velocity does not exist. The kinematic shift can be evaded by a sophisticated neo-Newtonian substantivalist. (Accounts of neo-Newtonian spacetime can be found in Sklar 1976, 202–206 and Geroch 1978, chap. 3.)

In sum, the Leibnizian arguments pose no insurmountable challenge to the substantivalist. Once God's creative act is removed from consideration, the PSR provides no grounds for objection to substantival space. The PII is a suspect metaphysical doctrine to begin with, and its weaker but more plausible cousin, the methodological maxim to avoid postulation of empirically undeterminable physical facts, cuts only against the kinematic and not the static shift. Finally, the kinematic shift and the accompanying notion of absolute velocity can be eliminated by adoption of neo-Newtonian spacetime rather than Newtonian absolute space.

4. Some New Arguments. At the end of the first section I remarked that a Newtonian relationist could account for inertial effects by postulating the same full set of spatiotemporal relations that obtain in absolute space but restricting their domain to the set of occupied points. Such a relationist could exploit the full resources of Newtonian dynamics because the fully specified set of distance relations between material events can be embedded in a Newtonian spacetime uniquely up to rigid rotations, rigid space and time translations, and parity. The relationist would regard this limited freedom in the embedding as arbitrary choices associated with establishing the origin and orientation of a reference frame. The different embeddings would be interpreted as representations of the same physical state, and the space in which the embedding occurs as a convenient fiction.[2]

In branding the different embeddings as representations of the same state the Newtonian relationist parts company with the substantivalist. Hence the relationist escapes from one of Leibniz's criticisms: The static Leibniz shift would not generate an ontologically distinct situation. The Newtonian relationist would, however, still fall prey to the kinematic shift. Since Newtonian relationism supports absolute velocities, the relationist would have to admit that there are facts about the universe that no experiment can determine. This, as we have seen, is methodologically distasteful.

Since the substantivalist can evade the kinematic shift by moving to neo-Newtonian spacetime, one naturally wonders if there could be a neo-Newtonian relationist ontology immune to both the static and kinematic shifts. Neo-Newtonian relationism, though, is a doctrine difficult to specify. It should posit a richer set of relations between events than does Leibnizian relationism, but not as full as Newtonian relationism. Neo-Newtonian spacetime does not have the complete four-dimensional metric provided by absolute space. It results rather from the addition to Leibnizian spacetime of an affine connection which (roughly) specifies which spacetime trajectories are straight, and hence inertial. So a reasonable version of neo-Newtonian relationism would add to the Leibnizian relations a three-place predicate col(x, y, z) which has as its extension all triples of nonsimultaneous collinear events.[3] That is, col(x, y, z) iff x, y and z all lie along some inertial trajectory. Specifying col(x, y, z) completely in a spacetime would fix the affine connection therein.

The relationist, however, is given as data only the restriction of col(x,

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[2] The idea that the relationist/substantivalist debate turns on construing spatiotemporal relations between material objects as embeddable in a fictional spacetime (relationism) as opposed to being a submodel of the model of all real spatiotemporal facts (substantivalism) was developed by Michael Friedman (1983, chap. 6). Friedman's main objection to relationism, though, strikes me as misguided. He claims that in order to formulate laws of motion one must make reference to inertial frames, and that while for the substantivalist such frames always exist, the relationist may find that no actual material objects travel on inertial trajectories, and so no inertial frames "exist". But the relationist does not need *occupied* frames to be able to state the laws of motion. If the spatiotemporal relations between material objects can be embedded uniquely (up to gauge freedom) into a fictional spacetime, then the laws can be formulated in terms of fictional inertial frames in that spacetime. Once the embedding is achieved, the relationist has all of the substantivalist's formal machinery.

[3] John Norton has sensibly asked whether col(x, y, z) is the *only* possible predicate that the relationist might use here. It is the only obvious one I could think of.

y, *z*) to the set of occupied points. This will, in general, be information too meager to specify an embedding of the occupied trajectories into the full spacetime up to the freedom associated with the choice of reference frame. For example, consider two particles in a neo-Newtonian spacetime that are uniformly rotating about their common center of mass. Until the first rotation is complete, no triple of occupied event locations are collinear. Even after any number of rotations, the collinearity relations among occupied points will be consistent with any periodic rotation, uniform or nonuniform. So while the Newtonian relationist could make use of the full power of Newtonian absolute space, starting only with the relations between occupied points, embedding them in a fictional space, and deriving via Newtonian mechanics exhaustive predictions about the future relations between occupied points, the neo-Newtonian cannot perform the same trick. Knowing, for example, that two particles remain at a constant distance through some period of time and that no three points in their world-lines are collinear, the neo-Newtonian relationist could not predict when, if ever, a triple of points would be collinear. Nor could inertial effects be predicted or explained, since the absolute acceleration cannot be inferred from the data.

The predictive and explanatory power of a relationist theory depends both on the nature of the spatiotemporal relations admitted and on the domain over which the relations are defined. Given a particle ontology, the substantivalist may have a more powerful theory simply because the domain of the relations forms a plenum. Thus it is not always possible, as it is for the Newtonian relationist, to skim off the important physical structure from a substantivalist theory (by restricting the relations to occupied points) and leave the spacetime behind.

Hartry Field (1985) has pointed out another advantage that accrues to the substantivalist in virtue of having a plenum of points in the ontology. We have been granting the relationist a very powerful piece of machinery, namely, a distance function defined over all pairs of simultaneous material events. This function, which yields a real number for every pair of contemporaneous points,[4] is unacceptable to Field as a primitive since it commits one to a fundamental direct relation between physical objects and numbers. Even those less squeamish than Field about Platonic abstract entities should welcome a reduction of this function to a sparser ideology. This reduction, Field claims, a substantivalist can give but a relationist cannot.

Field postulates as primitive relations only a part-whole relation; a be-

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[4] More realistically, the function would yield a real number for every ordered quadruple of simultaneous points $\langle x, y, z, w\rangle$, representing the ratio of the distance between x and y to that between z and w. This would add a scaling factor to the gauge freedoms mentioned above.

tweenness relation which holds for an ordered triple of events $\langle x, y, z\rangle$ iff they are collinear and x is between y and z; and a four-place congruence relation $xyCzw$ which holds iff x, y, z, and w are all contemporaneous events and the distance from x to y equals that from z to w (ibid., 49). From this stock of primitives the substantivalist can define a complete armory of distance ratio relations. For example, let "xyC_2zw" stand for "the distance from x to y is twice that from z to w". The substantivalist can define this relation from the primitives as follows: $xyC_2zw \leftrightarrow \exists u$ (u is a point, and u is between x and y, and $xuCuy$ and $uyCzw$) (ibid., 50). The same method can be used to define all rational ratios and, by a limit process, irrationals. The relationist cannot rely on such a definition of C_2 since in general there is no guarantee that the point midway between two particles will be occupied and so fall within the domain of the quantifier. After trying many different possibilities, including the use of modal locutions, Field concludes that the only way for a relationist to be assured of a relation like C_2 is to posit it as a primitive in addition to C, and similarly for all of the other ratios. Thus the relationist's primitive ideology swells monstrously in comparison with that of the substantivalist.

Field's difficulty is another which, although couched in terms of a debate between substantivalism and relationism, turns crucially not on the ontological analysis of space and time but on the existence or nonexistence of a plenum. The spatiotemporal structure used in the formulation of physical theories is most naturally defined over a connected manifold of points, not over the disjoint set of occupied points acknowledged in the relationist's particle ontology. The relationist's attempt to recover the full power of the mathematics through the postulation of a fictional spacetime will always appear tortured, and will often not work at all.

Even granting the congruence relation as a primitive, as Field does, concedes more to the relationist than the substantivalist need allow, for *distance between two points* is for the plenist a derivative geometrical notion: The primitive is rather *length along a path*. The distance between two points is defined as the extremal length of a path connecting the points. But the relationist with a particle ontology cannot hope to replicate this derivation, for the points over which the paths are defined will not, in general, exist. "A distance metric is a function which takes pairs of spatial points as its arguments and assigns a positive real number to each pair as its value. But this has geometrical interest only in that the points are regarded as the endpoints of some interval, path or arc across which the points are at a distance" (Nerlich 1976, 20).

The relationist may remain unmoved by this line of argument. After all, *something* must be taken to be a primitive. The substantivalist may take length along a path, or perhaps something yet further from which this can be defined, but one must stop sooner or later. What metaphysical principle demands that later is preferable to sooner?

The answer to this question is not, I think, to be found in a general principle which always demands further derivations of first principles, or motors for unmoved movers. The answer is rather to be looked for by a close examination in what the deeper structure is able to explain. For example, the relationist who takes the distance function as primitive must postulate not only the function but also certain constraints that govern it. One such constraint is the triangle inequality: For any three points x, y and z the distance from x to y plus the distance from y to z must be at least as great as the distance from x to z (the sum of the lengths of any two legs of a triangle must be as great as the length of the remaining leg). The relationist must demand that observed distance relations conform to this law, otherwise the relations cannot be embedded in a Euclidean space. But the constraint has no explanation in the relationist scheme; it is just a fortuitous law of nature.

The substantivalist who derives the distance relations, though, can explain the triangle inequality. Since the shortest path from x to y conjoined with the shortest path from y to z is a path from x to z, the shortest path from x to z (i.e., the distance) from x to z cannot be longer than the sum of the distances from x to y and y to z. One should postulate more structure, deeper structure, for the same reason one postulates any structure to begin with: to explain observable patterns in the phenomena. Discovering and accounting for regularities is the primary function of physical theorizing, and so long as complex regularities remain bare posits, the search for deeper explanatory structure should continue. This platitude militates a preference for the plenist ontology, which can explain the triangle inequality, over the nonplenist which cannot.

5. The Special Theory. Neo-Newtonian spacetime is an extraordinarily hostile environment for the relationist. The restriction of neo-Newtonian spatiotemporal structure to the occupied event locations of a particle ontology does not provide enough resources for either prediction or explanation of observable phenomena. In the arena of Minkowski spacetime, however, the relationist's fortunes are entirely reversed. (This form of relationism was noted and discussed by Earman 1989, 128–130. He comes to the same conclusions we do: Relationism of this sort works well in Special Relativity but comes to a disastrous end in General Relativity.)

The Minkowski relationist would posit only particles and the special relativistic spatiotemporal relations that exist between the events on their world-lines. The only plausible candidate for the spatiotemporal relation is the invariant interval between events, expressed in the usual coordinates as $I = \sqrt{c^2\Delta t^2 - (\Delta x^2 + \Delta y^2 + \Delta z^2)}$. When this function is fixed for every pair of occupied points the particle trajectories can be embedded in a Minkowski spacetime up to a global space and time translation, rigid rotations, Lorentz transformation, parity, and time inversion. For the relationist, these transformations, which leave all relations unchanged, are consequences of arbitrary conventions associated with setting up coordinate systems and reflect no real indeterminacy. Once so embedded, the relationist can make use of the full mathematical resources of substantivalist spacetime to make predictions about the future evolution of the relative quantities.

The embedding is so highly constrained because the relationist can use a triangularization procedure to fix the placement of points once a set of five nonco(hyper)planar points are placed. The method is simple. The first material point A can be embedded at random. The second, B, can be randomly chosen from the hypersurface of points with an interval I_{AB} from A. The third must lie on the intersection of two hypersurfaces: that of points I_{AC} from A and that of points I_{BC} from B. By the time the fifth point is placed the choice will be constrained to two possibilities—associated with the parity—and beyond that only one unique point will satisfy all the necessary constraints.

The Minkowski relationist, like the Newtonian relationist, can easily accommodate the bucket experiment. The special relativistic spatiotemporal relations between points making up a nonrotating bucket are different from those among the points in a rotating bucket. In particular, choosing parallel spacelike hyperplanes through the world-line of the bucket we will find that in the nonrotating bucket the world-lines of particles between the planes all have the same "length" while in the rotating bucket the particles near the axis will traverse a longer "distance" in spacetime than those along the edge.[5] Thus Mach's program, and the allied question of whether the General Theory satisfies Mach's principle, is moot. Special Relativity already resolves the problem of inertial forces for the relationist.

While Newtonian relationism seemed a highly artificial doctrine, infected with the specter of absolute velocities which could not be exorcized, Minkowski relationism is the only plausible form of relationism possible in the context of the STR. And like its predecessors, Minkowski relationism admits no metaphysical possibility of a static Leibniz shift, while the Minkowski substantivalist must regard such a shift as describing an ontologically distinct, physically possible state of affairs.

Furthermore, if any particle trajectory is inertial for any finite period

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[5] The "length" involved is the integral of the relativistic interval along the world-line. For timelike trajectories this is essentially a measurement of the proper time of a particle on that path. The fact that the particles at the edge of the bucket follow shorter trajectories is therefore due to the relativistic time dilation.

of time or if instead of idealizing particles as pointlike one assigns them a finite size, the Minkowski relationist can avoid the problem raised by Field for the relationist. As we saw above, Field claims that the relationist ideology must swell since the relationist cannot be sure that predicates such as "twice as far" can be defined in terms of the primitive congruence relation. This was because there is no guarantee that a material particle will exist halfway between any two given particles. As a result, merely specifying the extension of the congruence relation between particles does not much constrain the way those particles can be embedded in a Euclidean space. One must also specify the exact ratios of distances between particles, and each of those ratios must be, according to Field, postulated as a new irreducible predicate.

But if any finite stretch of an inertial trajectory is occupied (and within the world-line of a particle with nonzero volume will be some part of an inertial trajectory) then embedding of the part/whole, betweenness and congruence relations between material events into the Minkowski space-time will be unique up to the freedoms mentioned above. The proof is by construction. Suppose some finite length of an inertial trajectory is occupied. (We can identify it as inertial by the betweenness relation, since that relation only holds among collinear triples of points, and only along an inertial trajectory are all triples of points collinear.) The segment would form a small plenum which the relationist could use just as the substantivalist does. That is, taking the length of the segment as a unit, intervals of any length between 0 and 1 could be defined by Field's method. For example, the half unit can be defined as the length between an endpoint of the segment and the point equidistant from the two endpoints. Then an entire world-line could be embedded in the Minkowski space using the usual triangulation method by successively triangulating from points within a unit of the point being situated. This will be possible since the world-line is continuous, so successive points will always exist which are within a unit from already embedded points. Finally, with one complete world-line embedded one can calculate by the usual geometrical formulas the distances between any two points. Thus distance relations from 0 to the maximal length of the world-line will be fixed. Given the usual continuity constraints on matter, continuous trajectories will traverse the whole history of the universe.

Minkowski relationism is thus able to overcome every technical objection we have brought against relationism. Remaining are the more philosophical problems of understanding what these relations are (given that they are not measures of the extremal paths between points) and of explaining why the system of relations must always have the fortuitous form which allows it to be embedded into a fictional Minkowski space-time. But explanation and understanding must give out eventually, and perhaps these mysteries are not too high a price to pay for the privilege of banning substantival spacetime from one's ontology. After all, if the only proffered explanation of a phenomenon is spooks and goblins, perhaps it is best left unexplained.

6. The Final Round. The triumph of the Minkowski relationist under the regime of the Special Theory is startlingly ephemeral. Once one advances to the GTR, the relationist committed to a particle ontology inherits a hopeless task.

Ironically it is exactly the absoluteness of Newtonian space and of Minkowski spacetime that allows the relationist to treat them as fictions yet still exploit their mathematical utility. Because the geometrical structure of these spacetimes is fixed independently of the matter in them, one knows a priori the nature of the manifold into which the particle trajectories must be embedded. Furthermore, the geometrical structure, being fully metrical, is rich enough to support the triangularization procedure which can fix the position of points by means of a few distance relations to reference points already embedded. One need not, from the purely relational facts that the relationist admits as data, reconstruct the geometry of the embedding space, only the positions of the particles in it.

In the GTR, though, the spatiotemporal structure of the embedding space is not given a priori. Total information about the relations between material particles may not fix the structure of the whole spacetime sufficiently to permit prediction. For example, no amount of information about the past history of a set of particles can determine whether a gravitational wave is approaching from outside the system, a wave whose presence has not been recorded on any of the material world-lines. The material trajectories could be equally well embedded in fictional spacetimes with or without such gravitational waves, but the predictions derived from each model will differ, for the wave will disrupt the relations between particles.

The set of all spatiotemporal relations between occupied event locations cannot generally provide enough information to uniquely settle the geometry of the embedding spacetime. Consider a world whose material constituents are two particles that maintain a constant separation (i.e., the extremal interval from any point on one trajectory to the other trajectory is always the same). The relativistic interval between points on their world-lines is determined (from a substantivalist point of view) by the geometry of a thin sandwich of spacetime bounded by the world-lines, the sandwich within which all of the extremal paths between points lie. Little or nothing can be inferred about the geometry of the spacetime outside that sandwich. But the geometry of an entire spacelike hypersurface must be known in order to use the field equations to solve for the future evolution of the particles and of the relations between them. It is even a doubtful, though to my knowledge unexplored, proposition that the geometry of the volume in the sandwich can be recovered from the relational information.

Given these difficulties, the only hope for the relationist faced with the GTR is to abandon a particle ontology and adopt some form of plenism. The most natural way is to switch to a field ontology, a field which is nomologically constrained to be nonzero everywhere. Candidates for such a field will be considered below.

One might suspect that plenism plus relationism automatically results in a position identical to substantivalism, for the substantivalist too is working with a plenum of point entities bound together by spatiotemporal relations. But the views cannot generally be the same for they sometimes continue to disagree about the range of metaphysical possibilities. The Minkowski substantivalist, for example, must still admit that a static Leibniz shift generates an ontologically distinct state of affairs. One can move the fields and particles over in the spacetime to occupy different locations. The Minkowski relationist plenist, however, would deny any such metaphysical possibility. The two substantivalist models describe fields and particles with identical spatiotemporal relations between their corresponding parts, so the relationist would recognize no difference between them. The substantivalist's plenum is one which the fields occupy, but the relationist's is one which the fields constitute.

Even though substantivalism and plenist relationism are not per se mere notational variants, the difference between them becomes elusive to the point of evaporation in the context of the GTR. The static Leibniz shift depends for its application on the symmetries of the spacetime: Homogeneity is needed for translational shifts, isotropy for rotation. In general, though, the solutions of the general relativistic field equations are neither homogeneous nor isotropic so this means of distinguishing the two positions becomes useless. Those solutions which admit of symmetries do so because their matter and field distributions are symmetric, and in those cases the substantivalist will agree that the shift results in an unchanged ontological state. Rotating a radially symmetric field about its axis of symmetry, for example, results in the same field configuration even from the substantivalist point of view. Despite Clarke's intuition that God could have created a distinct but observationally indistinguishable situation by exchange of identical particles (see Leibniz 1956, 30), the substantivalist can deny that any principium individuationis exists that would allow a particle to retain its identity under a counterfactual operation that maps its world-line onto a trajectory that is presently occupied by an identical sibling. If *this* electron had had exactly the spatiotemporal career of *that* one (and vice versa) it would not have been this one. (Earman and Norton

1987 have contested the claim that no analogue of the static Leibniz shift applies to the GTR. See Maudlin 1989, 1990.)

While the static Leibniz shift cannot be formulated in any realistic (i.e., nonhomogeneous) General Relativistic spacetime, the kinematic shift cannot be formulated at all, since the spacetime structure does not admit of absolute velocities. So insofar as the debate is motivated by the considerations voiced by Leibniz and Newton, the substantivalist/relationist debate has at last resolved itself into a purely verbal dispute. According to the substantivalist the world is at base a manifold of spacetime points which support fields, in which the fields inhere. According to the plenist relationist the world is a field or set of fields (and perhaps particles) which instantiate spatiotemporal relations, in which the relations inhere. This difference of expression cannot, in the context of the GTR, be promoted into any further dispute about the physical or metaphysical facts.

7. Some Morals. Although we seem to be left with only one metaphysical position, differently expressed, it does not follow that the debate is a draw. Let us assess the final position of the antagonists.

The relationist has been forced to accept plenism, which at present entails accepting some sort of field ontology. Furthermore, the field must always be nonzero in order to avoid the problems caused by the vacuum. Contemporary physics gives relationists three choices here. First, they could claim that according to quantum field theory all fields are everywhere nonzero due to vacuum fluctuations. Second, they could maintain that a Higgs field has no classical vacuum state, that is, no state in which it does not exist, because it has no state characterized by both minimal energy and zero-field intensity. The Higgs "vacuum" is a structured entity: Something is there. Or third, they could adopt the metric field as the required object since its "vacuum" state is the Minkowski metric, not a zero tensor.

The first two alternatives are a bit embarrassing because physics does not yet tell us that the existence of any material field is necessary for the existence of spacetime. Admittedly this is because we have as yet no good idea of how quantum field theory and the GTR are to be combined, so possibilities of such an ultimate unification of spatiotemporal and material structure cannot be foreclosed. But as of now the GTR admits of vacuum solutions in which the stress-energy tensor is identically zero: No material exists. Indeed it admits of many such distinct solutions with different gravitational waves of space itself.[6] So the metric field is the most prom-

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[6] Since gravitational waves provide critical support for the substantivalist position, it may be useful to review the direct evidence we have for their existence. They have not been directly observed in the laboratory, but relevant astronomical evidence is available. Observations of a binary pulsar system have verified energy loss which matches the energy that should be carried away by gravitational radiation. See Will (1986, 201–206).

ising candidate to provide relationists with the needed plenum. But if they adopt this position it is hard to see how relationists have achieved any of their original aims. Whatever was objectionable about substantival space and time, their nonmateriality, unlikeness to middle-sized objects, and so on seems equally objectionable in the metric field. If relationists are willing to admit into their ontology a metric field, which fundamentally has only spatiotemporal properties, as a substance, why not just call it spacetime and join the ranks of substantivalists?

Conversely, substantivalists have had to abandon very little. Space alone can no longer be considered a substance as Newton envisioned it, with numerically identical parts subsisting through time. The structure of spacetime cannot be considered absolute, fixed independently of its material contents. But for all that, spacetime emerges as an entity in its own right, capable of a robust existence apart from particles and material fields. The conception of spacetime at which we have arrived is much more the heir of Newton than of Leibniz.

Two final observations are in order. First, the tumultuous career of nonplenist relationism suggests that it is not a hopeful doctrine given the mathematical formulation of physical theories. Theories couched in terms of local differential equations are most naturally interpreted as describing locally propagated effects, and so require some locality through which the effects can propagate. If the spacetime forms a fixed background in the theory, there may be some way to treat it as a convenient fiction, but seemingly minor changes in the theory can radically alter the success of such stratagems. Whether one is a relationist or a substantivalist, plenism is bound to be a more secure and resilient ontology than a particle ontology.

Second, if relationism and substantivalism have collapsed into the same view, this is for reasons of physics and not through some general metaphysical analysis. With every successive change in the physics, each approach acquired new strengths and weaknesses. So the issue is by no means closed; it must be reexamined in light of each new theoretical advance.

Even now attempts are being made to derive general relativistic spacetime from some more primitive, prespatiotemporal basis (see, e.g., Witten 1988, sec. 6). The deep ontology of physics twenty years hence may be as different from today's as today's is from that of Democritus. The pendulum may swing back to relationism or, more likely, the structures postulated may have such unfamiliar properties that the notions of substance and relation cease to have any clear application. But I leave future physics for future philosophers of science. *Il faut cultiver notre jardin.*

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