Is Scientific Metaphysics Possible?

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I. THREE KINDS OF METAPHYSICS

METAPHYSICS is back and doing rather well considering that it was pronounced defunct several times over the past two centuries. It comes in three different species: plain, exact, and scientific. Plain metaphysics ranges from elaborate nonsense through archaic common sense to deep and sophisticated yet outdated good sense. Although it is a rich mine of problems and insights, plain metaphysics is too far removed from contemporary knowledge. On the other hand, exact metaphysics, as exemplified by the so-called "calculus of individuals" and by some recent theories of possibility and of time, is done with the explicit help of contemporary logical or mathematical tools. It is living proof that metaphysics need not be obscure and on bad terms with logic. All too often, though, exact metaphysics neglects the philosophical tradition or does not care much for science, thus tending to become a futile exercise in applied logic. Scientific metaphysics is more ambitious: it attempts to solve some of the problems left over by plain metaphysics, it tackles new problems, and it tries to be in tune with both formal and factual science.

The purpose of this paper is to inquire into the possibility of this last alternative. But before we do so let us propose a more precise characterization of the object of our inquiry. A view may be said to constitute a piece of scientific metaphysics just in case it satisfies the following conditions:

(i) It concerns "the most general features of reality and real objects" (Peirce[1]) rather than ghosts.

[1] C. S. Peirce, *Collected Works*, 6.6, C. Hartshorne and P. Weiss, eds. (Cambridge, Mass.: Harvard, 1935). See also M. Born, *Physics in My Generation* (London: Pergamon Press, 1956), p. 94: metaphysics is "an investigation into the general features of the structure of the world and our method to deal with this structure."

(ii) It is systematic, i.e., a theory or part of a theory (hypothetico-deductive system) rather than an array of views.

(iii) It makes explicit use of logic or mathematics.

(iv) It is compatible and even continuous with the science of the day.

(v) It elucidates key concepts in philosophy or in the foundations of science.

(vi) It may be made to occur among the presuppositions of a scientific theory or it may develop into a scientific theory by specification or addition of specific hypotheses.

Condition (i) is necessary for all three kinds of metaphysics. (But it is not met by Thomas Aquinas', which, unlike Aristotle's, coincides with theology.) Conditions (i), (ii), and (iii) are jointly necessary and sufficient for exact metaphysics. The remaining conditions characterize scientific metaphysics by contrast to both plain and exact metaphysics. Consequently, if scientific metaphysics exists, then it encompasses the formally and scientifically sound parts of the other two.

The condition (i) of extreme generality makes it impossible for a metaphysical theory to issue definite predictions. Hence, *pace* Peirce,[2] metaphysics cannot be an observational science—except vicariously, i.e., through the intermediary of science. On this point we agree, then, with tradition. Moreover, by the same token, metaphysics is hardly refutable on empirical grounds alone, and so is any other set of theories that make no definite forecasts. This does not entail that metaphysics must be cut off from science: if it were, none of the conditions (iv) to (vi) could ever be met.

Although a metaphysical theory can be neither confirmed nor refuted by empirical data, it can be either relevant to science or pointless with regard to it. If relevant, it may eventually become a scientific theory upon being enriched with specific assumptions. Or it may be shown to constitute a presupposition (a part of the background) of some scientific theory. (Thus every scientific theory concerned with change of some kind presupposes some theory of time.) The metaphysical presuppositions of a scientific theory do not hang in mid-air: they are not free speculations, but come and go with the theory. If anything, scientific metaphysics is less free than mathematics; for it must pass not only the test of inner cogency, but also the test of consistency with science and the test of relevance to philosophy.

Having characterized and praised the class of theories in scientific metaphysics, we must show that it is not empty. But before doing

[2] Peirce, *op. cit.*, 6.5.

so it will be helpful to see how one can go about building theories in scientific metaphysics.

II. THEORY CONSTRUCTION IN SCIENTIFIC METAPHYSICS

Any means should be permitted in constructing a metaphysical theory as long as it leads to a good theory: pinching from another field, analogizing, extrapolating, looking for models of abstract theories, and of course inventing radically new ones. Here, as in science and in mathematics, there is no royal road, and theories are judged by their works not by their scaffoldings. However, it is useful to know what kind of dirt roads are open to us, for then we may try them according to our need or mood. Let us then be more explicit.

The main ways (not methods) in which theories can be got for scientific metaphysics seem to be the following:

(i) *Taking over from science or technology* without further ado or nearly so. Example: automata theory (section V).

(ii) *Adapting or generalizing an existing scientific theory.* Example: generalizing the algebra of chemical reactions to obtain a theory of analysis and synthesis (section IV).

(iii) *Endowing a ready-made mathematical formalism with a metaphysical content.* Example: converting ring theory into a general theory of juxtaposition and superposition (section III).

(iv) *Formalizing insights of plain metaphysics.* Example: building a general theory of qualitative change.

(v) *Overhauling theories in exact metaphysics.* Example: revising Whitehead's theory of space and time to render it consistent with relativity physics and manifold geometry, and freeing it from phenomenalist ingredients.

(vi) *Building fresh theories.* Example: constructing an exact theory of integrative levels.

In the following sections we shall illustrate the first three strategies. We shall start with a very simple example.

III. METAPHYSICS OUT OF MATHEMATICS: ASSEMBLY THEORY

Some mathematical theories are so many ready-made structures waiting to be assigned a metaphysical content. For example, probability theory, so overtaxed and so sterile in other branches of philosophy, provides not only an exact elucidation of the modal concepts but also an articulation of a number of ideas on possibility and on chance. The theory of groups of transformations is a convenient foil for a general theory of reversible changes. And category theory might prove to be a suitable basis for a general theory of qualitative change. A methodical search over the whole field of pure mathematics should come up with a large number of structures relevant to metaphysics. Let us exhibit one of them: ring theory, which we shall adopt as the formalism of assembly theory, or the general theory of juxtaposition and superposition as well as of parts and wholes.

First the intuitive idea. Two pencils, whether close by or not, constitute a third system resulting from the juxtaposition or physical sum of the given entities. On the other hand, chocolate milk results from the superposition or blending (but not synthesis) of two systems. We may designate juxtaposition (or physical addition) by '$\dot{+}$' and superposition (or physical multiplication) by '$\dot{\times}$'. These two operations can be regimented in different ways. The way that we shall investigate presently is to give a suitable interpretation of ring theory.

A ring is a structure $\mathcal{S} = \langle S, +, \cdot, 0, 1\rangle$, where S is a set, 0 and 1 are selected elements of S, and $+$ and $\cdot$ are binary operations on S, such that:

(i) $\langle S, +, 0\rangle$ is an abelian group under addition; i.e., S is closed under $+$, $+$ is associative and commutative in S, and every element x of S has an additive inverse $-x$ satisfying: $-x + x = 0$;

(ii) $\langle S, \cdot, 1\rangle$ is a monoid (semigroup with unit element) under multiplication; i.e., S is closed under $\cdot$, this operation is associative in S, and, for every x in S, $1 \cdot x = x$;

(iii) Multiplication distributes on both sides over addition; i.e., if x, y, and z are in S, then

$$\text{(a) } x \cdot (y + z) = (x \cdot y) + (x \cdot z)$$
$$\text{(b) } (x + y) \cdot z = (x \cdot z) + (y \cdot z)$$

The metaphysical interpretation we suggest is given by the following code:

Interpr (S) = The set of all systems
Interpr (0) = The null individual
Interpr (1) = The world
Interpr ($+$) = Juxtaposition ($\dot{+}$)
Interpr ($\cdot$) = Superposition ($\dot{\times}$)

These semantic formulas plus the preceding mathematical postulates constitute the axiomatic foundation of our assembly theory. The rest is definition or theorem. But before going into either it will be convenient to give an illustration. Let a, b, and c be samples of different liquids initially in separate containers. The resulting system is: $a \dot{+} b \dot{+} c$ with $a \dot{\times} b = a \dot{\times} c = b \dot{\times} c = 0$, and $a \dot{+} 1 = b \dot{+} 1 = c \dot{+} 1 = 1$. Now remove one of the systems, say a: the outcome is: $a \dot{+} b \dot{+} c \dot{-} a = b \dot{+} c$. Finally bring a back and mix it with both b and c. The resulting system is $(a \dot{\times} b) \dot{+} (a \dot{\times} c)$, in agreement with the distributive law. Note that the world has been "there" (mathematically: 1 was in S) all the time even while we restricted our attention to our three partial systems.

Assembly theory is the natural context for elucidating certain important metaphysical concepts occurring in every scientific theory dealing with complex systems. They are the concepts introduced by the following conventions.

Df. 1. A system z is *composed additively* of the systems x and y iff $z = x \dot{+} y$.

Df. 2. A system z is *composed multiplicatively* of the systems x and y iff $z = x \dot{\times} y$.

Df. 3. A system is *composite* iff it is composed either additively or multiplicatively.

Df. 4. A system is *atomic* iff it is not composite.

Df. 5. If x and y are systems, then x is a *part* of y iff either $x \dot{+} y = y$ or $x \dot{\times} y = x$. Symbol: $x \dot{\epsilon} y$.

Df. 6. Two systems x and y are *separate* iff their superposition is nought.

It is easily proved that the part-whole relation $\dot{\epsilon}$ introduced by Definition 5 is (unlike the membership relation) a partial ordering relation. Moreover, since $\dot{\epsilon}$ is defined in terms of juxtaposition (or of superposition), and since this relation holds among any two members of S, it turns out that we can prove the following:

THEOREM. The totality S of systems is partially ordered by the part-whole relation $\dot{\epsilon}$.

Some subsystems of S will be linearly ordered. Every such chain will begin with the null thing (0) and end up in the world (1). Note that the latter is not the same as the set S of all systems: the universe is an individual—a composite one rather than Parmenides', but not a set. What has the ring structure is not the world, i.e., 1, but the set S of all systems, which is also the set of all the parts of the world. (Do not mistake the set S of all the parts of the world for the family of subsets of S, i.e., the power set of S.) And note that the null individual (0) is not the same as nothingness (ϕ).

We shall not develop assembly theory any further in this paper, although ring theory could certainly be exploited to derive a number of metaphysical theorems. We close our exposé with a remark on existence. Our systems are bare individuals with no properties other than those of belonging to S and of juxtaposing and superposing. Consequently we may explicate "a exists" as "a is a system", i.e., "a belongs to S." That is, existence may be construed as a predicate —not a unary predicate though, but the complex set-theoretic predicate "ϵ." We may thus distinguish two concepts of existence: qualified and unqualified. The former is the concept elucidated in quantification theory, whereas unqualified existence is just membership in some set of systems which need not share any property other than that of belonging to that set. In assembly theory, which is unconcerned with any intrinsic properties, unqualified existence suffices.

Ours is not the only possible elucidation and systematization of the intuitive ideas of juxtaposition, superposition, and their cognates. To begin with there is the system of mereology proposed by Leśniewski[3] and explained by Leonard and Goodman.[4] It is fairly complicated, it is not one of the standard mathematical theories, and it does not accomplish much. For one thing it does not seem to supply a satisfactory formalization of the idea of superposition or interpenetration. Then, there is a previous theory of my own, consisting in a certain model of Boolean algebra.[5] Finally, further alternatives might be possible.

How do we choose among these various rival formulations of assembly theory? We must check which of them fits best the conditions stipulated in section I. All three alternatives explored so far satisfy the conditions (i) to (iii) for exact metaphysics; moreover, they also satisfy condition (iv). But the so-called calculi of individuals, or systems of mereology, do not fully satisfy condition (v) insofar as they seem hardly applicable to the mixing of fluids, superposition of fields, and other interpenetration processes. Consequently they cannot be used in the axiomatic reconstruction of scientific theories in which the superposition concept occurs. We are left with the proposed interpretations of Boolean algebra and ring theory. Of these, the latter seems preferable to me at the present moment if only because it does not require superposition to be commutative. (If $\dot{\times}$ is assumed to be commutative, then the set S of all things is a commutative ring.) This is advantageous because some interpenetration processes might not be commutative. (Recall Fisher's story of the lady who claimed that tea with milk does not taste the same as milk with tea.) And we want our assembly theory to be faithful to the real world, hence sensitive to examples and counterexamples drawn from science. For this reason, contrary to Scholz's belief,[6] metaphysics will not attain final certainty and freedom from controversy when coming of mathematical and scientific age. What can be hoped for is that in scientific metaphysics debates will cease to be erratic, ideological, and sterile, because desiderata,

[3] See B. Sobocinski, "Studies in Leśniewski's Mereology," *Polskie Towarzystwo Naukowe na Obczyźnie Yearbook* (London: 1954/55).

[4] H. S. Leonard and N. Goodman, "The Calculus of Individuals and Its Uses," *Journal of Symbolic Logic*, v, 2 (June 1940): 45–55.

[5] See my *Foundations of Physics* (New York: Springer-Verlag, 1967), ch. 2, sec. 5.

[6] H. Scholz, *Metaphysik als strenge Wissenschaft* (Darmstadt: Wissenschaftliche Buchgesellschaft, 1965), p. 139.

assumptions, and procedures become explicit and under the dual control of mathematics and science. The sandwiching between these two should enrich and discipline metaphysics without stifling it—if only because the covers of the sandwich do not cease to grow.

IV. METAPHYSICS OUT OF SCIENCE: ANALYSIS AND SYNTHESIS THEORY

Chemistry, the science of analysis and synthesis *par excellence*, may be taken as a source of inspiration for building a general theory of analysis and synthesis of individuals of any kind. This theory should go beyond assembly theory (section III) in that it should indicate explicitly the qualitative changes accompanying the association and dissociation of individuals. In other words, ours should be a theory of the transformations of natural kinds or species. Moreover, it should embody a generalization of the chemical atomic hypothesis, according to which only integral numbers are involved in analyses and syntheses. The theory to be sketched presently meets these requirements.

The intuitive point of departure is this. Let A, B, and C be natural kinds: species of elementary particles, chemical elements, molecular species, germ cells, body cells, or what not. Consider the following processes symbolized with the standard chemical notation:

(1a) *Synthesis or combination*: $mA + nB \rightharpoonup pC$

(1b) *Analysis or dissociation*: $mA \rightharpoonup nB + pC$

where the semiarrow symbolizes the process of becoming, and m, n and p are positive integers. Our task is to elucidate and systematize these ideas without restricting them to chemistry nor, indeed, to actual changes of the synthesis or of the analysis kinds. This is not just a question of jumping from chemistry to metaphysics: we must first improve the chemical notation, then bring to light the algebra of chemical reactions. These two tasks are accomplished by the foundations of chemistry.[7] The need for the first task is quite obvious, if only because the sign '+' in the previous formulas is ambiguous. Indeed, while in (1a) it stands for combination, in (1b) it stands for juxtaposition. The ambiguity disappears if the above schemata are rewritten as

(2a) $mA + nB - pC = \phi$ (*Synthesis*)

(2b) $mA - nB - pC = \phi$ (*Analysis*)

where ϕ designates the null natural kind. Now the inputs are assigned positive coefficients and the outputs negative coefficients. And every reaction, of either type, is written out as a linear combi-

[7] R. Aris, "Prolegomena to the Rational Analysis of Systems of Chemical Reactions," *Archive for Rational Mechanics and Analysis*, XIX (1965): 81, and "Some Addenda," *ibid.*, XXVII (1968): 356.

nation, with integral coefficients, of the natural kinds involved in it. Moreover, the only specific primitive concepts needed are seen to be those of natural kind, null natural kind, and combination. The axioms to follow specify and interrelate these concepts.

AXIOM 1. Every K_i, where i is a positive integer, is a set and represents a possible natural kind.

AXIOM 2. There is a finite number of possible natural kinds. Call it T.

AXIOM 3. (a) The structure $\mathcal{K} = \langle K, +, \phi \rangle$, where $K = \{K_i | 1 \leq i \leq T\}$, is a commutative monoid.

(b) If K_i, K_j, and K_n are in K, then $\ulcorner K_i + K_j = K_n \urcorner$ represents a possible combination of an individual of the kind K_i with another individual of the kind K_j to form a member of the species K_n.

AXIOM 4. The ith possible reaction R_i over the set $\{K_j\} \subseteq K$, with $1 \leq j \leq T$, is represented by the equation

$$\sum_j \alpha_i^j K_j = \phi, \quad \text{where} \quad \alpha_i^j \epsilon Z = \{0, \pm 1, \cdots\}$$

Before we write our fifth and last axiom we need some definitions. In stating them we shall dispense with niceties.

Df. 1. If n is a positive integer and K_i, a natural kind, then 'nK_i' abbreviates '$K_i + K_i + \cdots + K_i$' (n addends) and represents the synthesis of n individuals of the species K_i.

Df. 2. The reaction resulting from reversing the sign of the change matrix α of any given reaction R_i is called the *reverse reaction* $-R_i$.

Df. 3. In any reaction the natural kinds with positive coefficients are called the *reactants* and those with negative coefficients the *products*.

Df. 4. Let R_i be a reaction involving at least two natural kinds. Then R_i is called a *synthesis* (*analysis*) if it has a single product (reactant).

Df. 5. Let R_L be a set of reactions. Then any natural kind that occurs as a reactant but not as a product in R_L is called an *atomic species of level L*. Every other species is called a *molecular species of level* $L + 1$,

Now we are ready for out last basic assumption:

AXIOM 5. Let R_i and R_j be two reactions over a set $\{M_k\}$ of molecular natural kinds. Then the resultant $R_i + R_j$ of the two reactions is represented by

$$\sum_k (\alpha_i^k + \alpha_j^k) M_k = \phi$$

And our last convention:

Df. 6. A set of reactions is called *interacting* or *dependent* (alternatively, *summative* or *independent*) if any of them can (cannot) be decomposed as a linear combination of all the other changes.

The preceding axioms and definitions allow us to give rigorous proofs of a number of theorems concerning the structure of change. Among them the following:

THEOREM 1. Let R be the set of all reactions (analyses or syntheses). Then the structure $\mathcal{R} = \langle R, +, -, \phi \rangle$ is a commutative group.

THEOREM 2. Let R_D be the subset of R consisting of all dependent (interacting) changes of the analytic or of the synthetic type. Then the structure $\mathcal{R}_D = \langle R_D, Z, +, \cdot, \phi \rangle$ is a module over the ring Z of integers.

In plain words: the composition of two reactions is a third reaction, every synthesis is matched by an analysis, and interacting changes can be multiplied by numbers to yield further reactions. Results such as these can be multiplied *ad libitum* with the help of the theories of monoids, groups, and modules. We can also compare sets of changes, e.g., study the morphisms of analyses and syntheses on different levels. There is no end to the string of theorems: by turning mathematical, metaphysics becomes not only exact but also infinite.

V. METAPHYSICS OUT OF TECHNOLOGY: AUTOMATA THEORY

Automata theory has been with us for some time, waiting for metaphysicians to pick it up: it illustrates the first of the strategies for theory construction listed in section II, namely stealing. It is a rich new field originally conceived as a theoretical basis for computer science. But, since it has been developed mainly by mathematicians hardly interested in hardware, it has proved to be so extremely general as to cover all kinds of systems, regardless of their nature.

An automaton, as idealized by automata theory,[8] is a system susceptible to stimuli of a given kind. It jumps from one internal state to another in response to those inputs; and it can produce outputs depending on both the inputs and the internal states. Automata theory takes into account both the gross structure and the behavior of a system, but disregards the nature as well as the spatiotemporal arrangement of its components: it is a stuff-free, atopic, achronic, and grey-box theory. Hence it is applicable to all kinds of system in all kinds of environment and satisfying any laws compatible with the rather mild restrictions imposed by the definition of an autom-

[8] See, e.g., M. A. Arbib," Automata Theory," in R. E. Kalman, P. L. Falb, and M. A. Arbib, *Topics in Mathematical System Theory* (New York: McGraw-Hill, 1969); S. Ginsburg, *An Introduction to Mathematical Machine Theory* (Reading, Mass.: Addison-Wesley, 1962); A. Ginsburg, *Algebraic Theory of Automata* (New York: Academic Press, 1968); or M. A. Harrison, *Introduction to Switching and Automata Theory* (New York: McGraw-Hill, 1965).

aton. In particular, automata theory is applicable to neuron assemblies and to societies, not only to artifacts.

In other words, the referent of automata theory is a system-environment compound of any nature whatever—mechanical, electrical, chemical, biological, or behavioral—and subject to the following limitations. First, an automaton admits inputs of a single kind, e.g., punched cards, and its outputs are likewise of a fixed kind, e.g., printed symbols. The simplest automaton can print just what it can "read," e.g., strings of zeros and ones. Second, a (finite) automaton can be in either of a finite number of states. Third, an automaton can go through certain sequences of states only, and it does so in a discontinuous manner: it is a discrete and sequential system.

Automata theory is of interest to metaphysics, nay, part and parcel of scientific metaphysics, precisely because (a) it provides a rough conceptual model of a thing interacting with its environment regardless of any specific features of interest to the particular sciences, and (b) it is stuff-free, atopic, and achronic, hence independent of any law statements—nevertheless concerned with some traits of concrete entities. To persuade ourselves that this is the case we proceed to expound our own version of the axiomatic foundations of the theory of deterministic automata. The only departure from standard exposés will be found in the axiomatization style, which intends to exhibit not only the form but also the content of the basic concepts. (This kind of axiomatics has been characterized elsewhere.[9])

The specific primitive or defining concepts of the theory are listed in the following table.

Symbol	*Mathematical nature*	*Factual content*
Σ	Set	Collection of distinct unit inputs
Ω	Set	Collection of distinct unit outputs
Λ	Individual	Null input
$\circ$	Binary operation	Concatenation of inputs or outputs
S	Set	State space
M	Function	Transition (next-state) function
N	Function	Output function
s_0	Individual	Initial state
F	Set	Collection of final states

[9] See my "Physical Axiomatics," *Reviews of Modern Physics* (1967): 463, and "The Structure and Content of a Physical Theory," in Bunge, ed., *Delaware Seminar in the Foundations of Physics* (New York: Springer-Verlag, 1967).

The automaton concept is characterized by the following axiomatic:

DEFINITION. The structure $\mathfrak{A} = \langle \Sigma, \Omega, \Lambda, \circ, S, M, N, s_0, F \rangle$ is said to represent a *finite deterministic automaton* (= sequential machine with output) A if and only if

A1a. $\Sigma = \{\sigma_0, \sigma_1, \cdots, \sigma_{k-1}\}$, called the *input alphabet*, is a nonempty set with k elements called *letters*.

A1b. Every $\sigma_i \epsilon \Sigma$, for $i = 0, 1, \cdots, k-1$, represents a unit input on A from its environment.

A2a. Ω, called the *output alphabet*, is a set with two elements: 0 and 1.

A2b. $0 \epsilon \Omega$ represents the absence of an output of A; $1 \epsilon \Omega$ represents the reaction of A to its environment.

A3a. Let Σ^* be the set of finite concatenations of elements of Σ, and let $\Lambda \epsilon \Sigma^*$ be such that, for every $x \epsilon \Sigma^*$, $\Lambda x = x \Lambda = x$. Likewise let Ω^* be the set of finite concatenations of elements of Ω, with $0 \epsilon \Omega^*$ and such that, for every $y \epsilon \Omega^*$, $0y = y0 = y$. Then the structures $\langle \Sigma^*, \Lambda, \circ \rangle$ and $\langle \Omega^*, 0, \circ \rangle$ are monoids.

A3b. $\circ$ represents the combination or concatenation of successive inputs or of outputs of A.

A4a. S, called the *state space* of A, is a finite nonempty set with n elements.

A4b. Every $s_i \epsilon S$, for $i = 0, 1, \cdots, n-1$, represents an internal state of A.

A5a. M, called the *transition* (or *next-state*) function, is a function from the cartesian product $S \times \Sigma^*$ into S.

A5b. If $s \epsilon S$ represents a state of A and $x \epsilon \Sigma^*$ represents an input (word) on A, then $M(s,x) \epsilon S$ represents the state A goes into when x is applied to A while in state s.

A6a. N, called the *output function*, is a function on $S \times \Sigma^*$ into Ω^*, such that $N(f,x) = 1 \epsilon \Omega^*$ for every $f \epsilon F \subseteq S$ and every $x \epsilon \Sigma^*$.

A6b. $N(f,x)$ represents the output of the automaton when acted on by the input $x \epsilon \Sigma^*$.

A7a. s_0 is in S.

A7b. s_0 represents the initial internal state of A.

A8a. F is a nonempty set included in the state space S.

A8b. Any member f of F represents a final state of A.

A9. A makes no spontaneous transitions; i.e., the null input has no effect: For every s in S, $M(s,\Lambda) = s$.

A10. Internal states form sequences; i.e., the effect of a compound input $x \circ y$ formed from $x, y \epsilon \Sigma^*$, equals the effect of the second input acting on the automaton in the state to which the first input carried it:

$$M(s,x \circ y) = M(M(s,x),y)$$

The preceding axioms are necessary and sufficient to characterize a deterministic automaton with a finite number of states. However, they do not determine precisely the central properties of A, namely, the transition function M and the output function N. Thus A6a is a disguised definition of a final state, for it amounts to saying that a final state is any state which, acted on by an arbitrary input, gives rise to a one output. As to A9 and A10, they only restrict the possible transition functions M to those characterizing systems which operate (a) by external compulsion, (b) serially (sequentially), and (c) in a determinate (or nonprobabilistic) way. Although the last two axioms thus characterize the type of transition function, they do not specify the function exactly—nor are they supposed to, since they should be elastic enough to accommodate all kinds of finite, sequential, and deterministic systems. In other words, A9 and A10 are not law statements but rather conditions (among others) that a system must satisfy in order to *qualify* as, or to deserve being *called*, a finite deterministic automaton—or, better, a causal sequential system. It is up to the applied automata theorist (be he an engineer, a psychologist, or a linguist) to manipulate real components (and the laws inherent in them) so that a system results that will satisfy the conditions defining an automaton. If a real system, whether natural or artificial, physical or social, fails to satisfy any of the above ten axioms, it will not count as a falsifier of automata theory. In other words, it is impossible to refute automata theory. On the other hand, it is possible to confirm or illustrate it by exhibiting systems behaving like automata in some respects. Could anything more metaphysical be imagined?

VI. CONCLUDING REMARKS

Theories in scientific metaphysics, just like scientific theories, pose three sets of technical problems: questions concerning form, content, and evidence. More explicitly, we must face the following questions:

(i) *Form.* Since scientific metaphysics is exact, a theory in scientific metaphysics should have a definite mathematical structure. Not necessarily a quantitative one, though: logic and algebra are most often sufficient. And even if numerical functions are involved in scientific metaphysics (as will be the case with a theory of space/time utilizable in the axiomatic foundation of a scientific theory), no laws or rules for computing special values of such functions need or indeed should be given. For, if they were given, then metaphysics would be able to compute definite predictions, thus competing with science. And metaphysics, far from competing with science, should cooperate with it.

(ii) *Content.* Scientific metaphysics, just like factual science, concerns the actual world rather than all logically possible worlds. Moreover, a theory in scientific metaphysics may concern either a general trait of the world, like change, or a trait peculiar to a fragment of it, like mental activity. More precisely, scientific metaphysics consists of two sets of theories: (a) *universal* or cross-level theories, and (b) *regional* theories limited to some integrative levels. However, even the most special theory in regional metaphysics will not be specific enough to be able to account for the details of any particular individual. If it does, then one more theory is gained for science. In any case, whether universal or regional, a theory in scientific metaphysics should include designation rules stipulating what its symbols denote, as well as semantic assumptions linking its basic concepts with definite traits of the referents of the theory.

(iii) *Evidence.* Since theories in scientific metaphysics are not specific or definite, they cannot be tested through prediction. In other words, they are not empirically testable. (In this respect they are not worse off than the most general scientific theories prior to the adjunction of specific assumptions and data.) But checked they must be if they are not to be sheer dogmas. The evidence for a theory in scientific metaphysics consists of judgments about its ability to (a) cohere with science, (b) elucidate and systematize philosophical concepts and principles, and (c) serve science by polishing up some of its metaphysical concepts (e.g., those of event and chance) and metaphysical hypotheses (e.g., those of lawfulness and of the rooting of the higher integrative levels on the lower ones). In short, the test of scientific metaphysics is science.

This last point deserves a comment. The articulation of science with metaphysics is not rigid: one and the same scientific theory may be consistent with alternative metaphysical theories. Thus we saw in section III that different assembly theories are possible and that the choice among them is by no means obvious. Another example: classical mechanics is by no means married to an absolute theory of space and time, but can be built on the basis of a relational theory. Consequently a scientific victory does not entail the triumph of any given metaphysical theory in tune with it: rather, it supports a whole class of metaphysical theories. The best among the latter will be those fitting in with most of science and supplying the most exact and cogent elucidation of the metaphysical ideas concerned.

To conclude. We have exhibited three theories, or rather their axiomatic foundations, that satisfy the conditions defining scientific metaphysics. We have thereby proved that scientific metaphysics is not only possible but actual. It is particularly alive in engineering schools, although it is doubtful that engineers will be flattered by this association. Nor is the scientist apt to feel happy if told that every scientific theory needs some metaphysics to take care of its generic metaphysical concepts and principles. He may well retort that he can dispense with such help, for he can take care of those items himself: for instance, if provoked the scientist can build mathematical theories elucidating and systematizing the generic concepts of system and of environment. He can indeed: he can turn metaphysician.[10]

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BOOK REVIEWS

Selected Works in Logic. THORALF SKOLEM. Edited by JENS ERIK FENSTAD. Oslo: Universitetsforlaget, 1970. 732 p. $27.50.

This important volume contains reprints of almost all of Skolem's papers in mathematical logic, papers spanning the period 1913–1963. The volume also contains an English translation of an early paper on Schröder's algebra of logic, a biographical sketch of Skolem by the editor, and a long, extremely interesting introduction by Hao Wang describing Skolem's contributions and outlining the main proofs. Finally, there is a complete bibliography of Skolem's writings, with a very useful listing of the principal reviews of them. Skolem's major papers, from which stems his profound influence on mathematical logic, appeared between 1920 and 1934; these papers contain proofs of the two forms of what is now called the *Skolem-Löwenheim theorem,* discussions of the implications of this theorem for quantification theory and set theory, the development of recursive arithmetic, and the construction of a nonstandard model of arithmetic. Skolem's later work for the most part comprises investigations of more specific problems in lattice theory, recursion theory, recursive arithmetic, decision problems, and variant set theories.

[10] For a detailed discussion of the continuity between metaphysics and science (i.e., the lack of a clear demarcation line between them) see my "Testability Today," forthcoming.